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# WCET Free Time analysis of Hard Real-Time Systems on Multi-Processors: a Regular Language-Based Model

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## Abstract

This paper presents the initial step of an aid design method earmarked for operational validation of hard real-time systems. We consider systems that are composed of sequential hard real-time jobs, which are embedded on centralized multiprocessor architectures. We introduce a model based upon untimed finite automata and meant to collect the operational behaviours of the system compatible with its time specifications, and we go on to provide a feasibility decision result for systems composed of jobs presenting CPU-loads which are exact values: execution times are not WCET values). This is why we call this approach *WCET-free analysis*. The results we have achieved likewise involve hardware specifications such as multi-processors and speeds of processors.

*Key words:* Finite automata, Real-Time Systems, Operational validation

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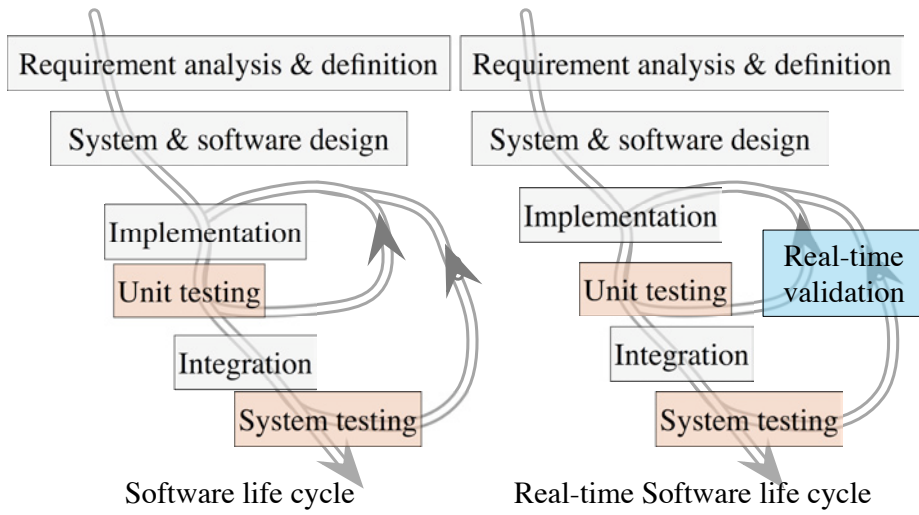


Figure 1. The operational validation in the software life cycle

## 1 Introduction

Real-Time scheduling has been implemented in real-time kernels by means of fixed priority policies (mainly RM and DM). Dynamic priority policies (EDF, LLF) also exist [26], but they have not been implemented in operating systems. The scheduling power of all these policies has been studied for uniprocessor and multiprocessor targets [27][15]. Since they have not been designed to deal with precedence or synchronization constraints, additice specific techniques have been designed [31][8].

Real-time validation has been studied: it consists in deciding whether specific software can be scheduled on a given target (feasibility) or if a specific on-line policy can be used to schedule the software (validation). The knowledge of all valid scheduling sequences concerning a task system designed to run on a specific hardware target is useful (in particular) to chose the best scheduling technique which addresses the case study.

Actual *system* approaches are software-engineering oriented: for instance, [2] is designed to build specific policies or controllers, others are based on a categorization and the analysis of system constraints (global vs local in [2]), [32] integrates the real-time validation step into the software life cycle (see Figure 1), [11] proposes ACSR algebra, whose purpose is to facilitate conception and analysis into the specification step of the life cycle, etc. A more controler-oriented technique is proposed by the TIMES project [5]: producing a scheduler from the time specifications and calculating the worst response times of software tasks.

As far as we know, the problem of collecting all valid scheduling sequences

has not been studied for multiprocessor targets. Solving this problem will be useful to help the user to choose a good scheduling technique. It is the aim of this paper.

Previously existing feasibility studies (they address uniprocessor targets) stand on modeling scheduling sequences, like timed automata [4][3], Petri nets [21], etc. [34][29]. A comparative analysis between the capacity of untimed and timed automata [6][4][3] leads us to use of untimed finite automata: for this model, concerning the properties we deal with, the class of decidable properties is the largest, the computing complexities are not worse [3][16].

Each task of the system is translated into a finite automaton whose accepted words correspond to all possible behaviours of the task. The automaton is computed on the basis of the time characteristics of the task, in order to accept only the task behaviours compatible with its time constraints. Concurrency is modeled by products of automata, and synchronization (processor and resource sharing, communication) thanks to the Arnold-Nivat model [6].

Since we address the problem of collecting all possible behaviours of the systems we study, we do not deal with the *WCET* assumption [14], this is why we call our approach *WCET-free analysis*. In [23], Krčál and Yi show the decidability of the scheduling problem for systems specified by the  $[Min, Max]$  CPU loads. Amnell and al. cite this problem to be addressed [5]. In this paper, we are presenting a hopefully quite effective solution: we deal with systems specified by all possible CPU loads for all jobs of the system, and we are proposing an algorithmic technique based on untimed automata to reach this decision. Our technique revolves around solving the reachability problem, whose complexity is known to be lower for untimed automata. All these factors have got to express our choice on untimed finite automata (i.e. regular languages) to model real-time systems.

We have organised this paper as follows.

- Since scheduling decisions depend on both the analysed task system and the hardware and software context (see [22] for a complete analysis which points this fact out), the first part of the paper (Section 2 deals with the task systems we deal with and a taxonomy of possible contextes, in order to define a precise canvas for following studies.
- The second step consists in the technicals of the work: we present the basic elements of our language-based technique: Section 3 deals with the language based model and the validation process; Section 4 shows how to extend this technique to address synchronization; Section 5 shows how the *WCET* assumption can be avoided.
- The final section (§7) deals with the internals of the computing techniques and the results of experimentations to point out the improvement level

brought by our optimization technique.

## 2 Validation of real-time systems

In this section, we describe both the structure of real-time systems and a model meant to represent execution contexts.

### 2.1 Real-time systems

A real-time system is composed of a finite set  $(\tau_i)_{i \in [1, n]}$  of jobs. Each job  $\tau_i$  is a sequence of tasks (or instances)  $\tau_{ij}$  ( $j \in \mathbb{N}$ ). Each task  $\tau_{ij}$  is activated by the occurrence of an incoming event. When the flow of incoming events associated with  $\tau_i$  is periodic (the time interval between two successive incoming events is a constant  $T \in \mathbb{N}^*$ ),  $\tau_i$  is called periodic, and  $T$  is one of its characteristics. If not,  $\tau_i$  is called sporadic.

When all jobs of a real-time system are periodic, the system is likewise called periodic.

Each job  $\tau_i$  which belongs to a real-time system is time-specified with 4 time characteristics  $(r_i, T_i, C_i, D_i)$ . If  $\tau_i$  is periodic, the semantics of these characteristics are as follows:

$T_i$  is the period of  $\tau_i$ : it is the constant delay that separates the activation dates of two consecutive tasks  $\tau_{ij}$  and  $\tau_{ij+1}$ .

$r_i$  is the first activation date: it is the occurrence date for the first event of the flow associated with  $\tau_i$ . The activation date of a task  $\tau_{ij}$  is  $r_i + jT_i$ .

$C_i$  is the CPU load of tasks  $(\tau_{ij})_{j \in \mathbb{N}}$ : it is the CPU time that must be allocated to  $\tau_{ij}$  to complete its execution. Here, we consider the context of invariable CPU times for jobs: all tasks  $\tau_{ij}$  share the same CPU load  $C_i$ . In Section 5, we extend this model to address tasks whose successive instances do not share the same CPU load.

$D_i$  is the critical delay of  $\tau_i$ : it determines the deadlines of all tasks  $(\tau_{ij})_{j \in \mathbb{N}}$ , that are the dates when  $\tau_{ij}$  must be completed. The deadline for task  $\tau_{ij}$  is  $r_i + jT_i + D_i$ . In this paper, we assume that  $D_i \leq T_i$ .

It is impossible to specify at what exact time the initial activation event of an *alarm job* will occur: if  $\tau_i$  is sporadic,  $r_i$  may be undefined, and  $T_i$  indicates the minimum delay that separates two successive activations of  $\tau_i$ .

Scheduling a real-time system consists not only in attributing CPU allocation time to jobs, but also in doing so according to the deadline constraints. The feasibility of scheduling a real-time system most notably depends on the hardware framework (number of processors, distribution, etc.).

A scheduling context provides an accurate description of the hardware and software configuration the scheduling algorithm is supposed to work with. A syntax meant to describe these contexts has been proposed in [19], and has been extended in [9] and [20]. Here, we use a syntax adapted from that of [20].

An execution context is specified by a table

| Hardware     |             |               | Software       |                    |              |
|--------------|-------------|---------------|----------------|--------------------|--------------|
| Architecture | Clock       | Communication | Structure      | Synchronisation    | Preemption   |
| $(n, p)$     | <i>Mode</i> | $(Cpx, Snc)$  | $(Jb, St, Ld)$ | <i>Constraints</i> | <i>Prmpt</i> |

that can be understood with the following semantics:

- **Hardware specifications**

- **Architecture** =  $(n, p) \in (\mathbb{N}^* \cup \{n\})^2$   
 $n$  specifies the number of nodes, and  $p$  the maximum number of processors on each one.
- **Clock** =  $Mode \in \{common, harmonic, synch, independent\}$   
specifies the time dynamics of the processors of each node. Its specification is based on *unit* (the common duration of all atomic statements) and *start* (starting date of the processor driving clock). Using processors  $(p_i)_{i \in [1, p]}$  on a node, the semantics to be associated with the possible choices are the following:

**common** all processors follow the same clock:

$$(i, j) \in [1, p]^2 \Rightarrow \begin{cases} speed(p_i) = speed(p_j) \\ start(p_i) = start(p_j) \end{cases}$$

**harmonic** speeds of processors follow the same clock, in a harmonic way:

$$(i, j) \in [1, p]^2 \Rightarrow \begin{cases} speed(p_i) \in speed(p_j) \mathbb{N} \vee speed(p_j) \in speed(p_i) \mathbb{N} \\ start(p_i) = start(p_j) \end{cases}$$

**synch** all processors follow clocks which start simultaneously:

$$(i, j) \in [1, p]^2 \Rightarrow start(p_i) = start(p_j)$$

**independent** processors follow different clocks, and then there is no relation between speeds and starts

- **Communication** =  $(Cpx, Snc) \in \{H^0, H^-, H^+\} \times \{s, b, a\}$

**Cpx** specifies communication complexities:  $H^0$  for *centralized* (and then a null complexity),  $H^+$  for *non-null same complexity communications*,  $H^-$  for *heterogeneous communication*.

**Snc** specifies the synchronism level of communication channels. Channels between two tasks are implemented on buffers of size  $k \in \bar{\mathbb{N}}$ , that are specified in the following way: **s** for *synchronous* (i.e.  $k = 0$ ), **b** for *asynchronous with limited buffer* (i.e.  $k \in \mathbb{N}^*$ ), **a** for *asynchronous* ( $k = +\infty$ ).

- **Software specifications**

- **Structure** =  $(Jb, St, Ld) \in \{s, p, a\} \times \{0, r_i, \perp\} \times \{C_f, C_v\}$

with the following semantics:

- s** All tasks are sporadic

- p** All tasks are periodic

- a** There are periodic and sporadic tasks

- 0** The first instances of all tasks are activated synchronously at time 0

- $r_i$  The first instances of all tasks are not activated synchronously at time 0

- $\perp$  No specification is given on first activation dates

- $C_f$  All instances of a job require the same CPU allocation time

- $C_v$  Two different instances of a same job may require different CPU allocation times

- **Synchronisation** =  $Constraints \in \mathfrak{P}(\{r, c, p, x\})$

with the following semantics:

- r** There is resource sharing between jobs

- c** There is communication between jobs

- p** There are precedence constraints between jobs

- x** There are exclusion constraints between jobs

- **Preemption** =  $Prmpt \in \{notp, parp, totp\}$

with the following semantics:

- notp** Tasks cannot be preempted

- parp** Tasks can be preempted, excluding some specific contexts (e.g. critical sections)

- totp** Tasks can be preempted whatever the context

### 3 Time validation process for periodic systems

In this section, we consider the context

| Hardware     |               |               | Software        |                 |             |
|--------------|---------------|---------------|-----------------|-----------------|-------------|
| Architecture | Clock         | Communication | Structure       | Synchronisation | Preemption  |
| $p \geq 1$   | <i>Common</i> | $(H^0, s)$    | $(p, r_i, C_f)$ |                 | <i>totp</i> |

This context is called  $ICCT(p)$  in the following ( $I$  for *independent tasks*, the first  $C$  for *common clock*, the second  $C$  for *centralized system*,  $T$  for *periodic real-time system* and  $(p)$  for  $p$  processors). In the following, the task system  $(\tau_i)_{i \in [1, n]}$  is noted  $\Gamma$ :  $n$  is the number of tasks.

### 3.1 Basic notions on languages

The reader is presumed to be familiar with the basic notions about words, languages and automata [16]. However, we wish to recall two basic notions and some notations and results, of which we have made use in this paper.

In the following,  $Reg(\Sigma)$  is the class of regular languages on the alphabet  $\Sigma$ .

**Definition 1** Let  $\Sigma_1$  and  $\Sigma_2$  be two alphabets. The Shuffle ( $\text{III}$ ) is a binary operation on words or languages, defined in the following way:

- (1)  $\forall a \in \Sigma_1 \cup \Sigma_2, a \text{III} \epsilon = \epsilon \text{III} a = \{a\}$
- (2)  $\forall (a, b, \omega, \xi) \in \Sigma_1 \times \Sigma_2 \times \Sigma_1^* \times \Sigma_2^*, a\omega \text{III} b\xi = a(\omega \text{III} b\xi) \cup b(a\omega \text{III} \xi)$
- (3)  $\forall L_1 \subset \Sigma_1^*, \forall L_2 \subset \Sigma_2^*, L_1 \text{III} L_2 = \bigcup_{(\alpha, \beta) \in L_1 \times L_2} (\alpha \text{III} \beta)$

**Proposition 1** Let  $\Sigma_1$  and  $\Sigma_2$  be two alphabets, and  $L_1 \subset \Sigma_1^*$  and  $L_2 \subset \Sigma_2^*$ . We get  $L_1 \in Reg(\Sigma_1) \wedge L_2 \in Reg(\Sigma_2) \Rightarrow L_1 \text{III} L_2 \in Reg(\Sigma_1 \cup \Sigma_2)$

**Definition 2** Let  $L \subset \Sigma^*$ , and let  $\omega \in L$ . We call prefix of  $\omega$  every  $\alpha \in \Sigma^*$  such that  $\exists \beta \in \Sigma^*$  such that  $\omega = \alpha\beta$ .

The set of prefixes of a word  $\omega$  is denoted  $Pref(\omega)$ . This definition is extended to languages by way of  $Pref(L) = \bigcup_{\alpha \in L} Pref(\alpha)$ .

**Definition 3** Let  $L \subset \Sigma^*$ , and  $\omega \in L$ . We call infinitely extendable prefix of  $L$  every  $\alpha \in \Sigma^*$  such that  $\forall n \in \mathbb{N}, \exists \beta \in \Sigma^*$  such that  $|\beta| > n$  and  $\alpha\beta \in L$ . The set of infinitely extendable prefixes of  $L$  is called  $Center(L)$ .

**Proposition 2** We obtain

- (1)  $Center(L) = L^* Pref(L)$
- (2)  $L \in Reg(\Sigma) \Rightarrow Center(L) \in Reg(\Sigma)$



We define the notion of time unit in the following way: it is the execution time of an atomic statement (i.e. an assembler statement) on the target machine. In this work, we consider this value to be shared by all atomic statements.

Let us consider the job  $\tau_i$ , whose time parameters are  $r_i \in \mathbb{N}$ ,  $C_i \in \mathbb{N}^*$ ,  $D_i \in \mathbb{N} \cap [C_i, +\infty[$  and  $T_i \in \mathbb{N} \cap [D_i, +\infty[$ . From its activation date  $r_i + k \times T_i$ , the  $k^{th}$  instance of  $\tau_i$  must own a CPU resource for  $C_i$  time units on the time interval  $[r_i + k \times T_i, r_i + k \times T_i + D_i[$ . Let us note  $a_i$  the state  $\tau_i$  owns a CPU for one time unit, and  $\bullet$  the state  $\tau_i$  does not own a CPU for one time unit. Every word of  $a_i^{C_i} \text{III} \bullet^{D_i-C_i}$  corresponds, on any time interval of the form  $[r_i + k \times T_i, r_i + k \times T_i + D_i[$ , to a processor time allocation compatible with the time constraints of  $\tau_i$ . This set is regular. If the scheduling configuration is valid,  $\tau_i$  is inactive on every time interval of the form  $]r_i + k \times T_i + D_i, r_i + (k+1) \times T_i[$ . This inactivity is modeled by the word  $\bullet^{T_i-D_i}$ . Then, every word of  $(a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i}$  is a correct CPU time allocation for  $\tau_i$  on any time interval of the form  $[r_i + k \times T_i, r_i + (k+1) \times T_i[$ .

The task  $\tau_i$  is defined as the sequence  $(\tau_{ij})_{j \in \mathbb{N}}$  of its instances. A processor allocation compatible with  $\tau_i$ 's time constraints is a sequence of processor allocations compatible with  $\tau_i$ 's successive instance time constraints. Let  $(\omega_j)_{j \in \mathbb{N}} \in ((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})^{\mathbb{N}}$ . For each  $n \in \mathbb{N}$ , the word  $\omega_0 \omega_1 \dots \omega_n$  models a time valid processor time allocation for any sequence of  $n+1$  successive instances of  $\tau_i$ . In a general way, any word  $\omega$  of  $((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})^*$  models a valid processor time allocation for  $\tau_i$  on any time interval of the form  $[r_i + k \times T_i, r_i + k \times T_i + |\omega|]$ , and then, in particular, on the time interval  $[r_i, r_i + |\omega|]$ . Insofar as  $\tau_i$  is inactive on interval  $[0, r_i[$ , word  $\bullet^{r_i} \omega$  models a valid processor allocation on the time interval  $[0, r_i + |\omega|]$ .  $\omega$  can be as long as we wish. Then, the language  $\bullet^{r_i} ((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})^*$  collects all the valid processor allocation for  $\tau_i$ .

The scheduling validation problem consists, at time  $t$ , in deciding on the evolution possibilities of  $\tau_i$  in both a given hardware and a given software context. Of course, the past of  $\tau_i$  is known. Here, this past is the history of  $\tau_i$ 's CPU allocations, that is to say a finite word  $\omega$  of  $\{a_i, \bullet\}^*$ . During this past, some instances of  $\tau_i$  were completed, and the current instance is on (we can say so even at the outset). Then, by construction,  $\omega$  is of the form  $\omega_1 \mu$ , where  $\omega_1 \in \bullet^{r_i} ((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})^*$  (the completed past instances), and  $\exists \nu \in \{a_i, \bullet\}^*$  such that  $\mu \nu \in ((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})$  (the current instance can be completed according to the time constraints). So,  $\omega$  is a prefix of a word of  $\bullet^{r_i} ((a_i^{C_i} \text{III} \bullet^{D_i-C_i}) \bullet^{T_i-D_i})^*$ . Moreover, by construction as well, any instant

$f$  belonging to  $]t, +\infty[$  (the future), there exists  $\eta \in \left( \left( a_i^{C_i} \text{III} \bullet^{D_i-C_i} \right) \bullet^{T_i-D_i} \right)^*$  such that  $|\omega\nu\eta| > f$ . Following which, at each time  $t$ , the past  $\omega$  of  $\tau_i$  is a word of the center of  $\bullet^{r_i} \left( \left( a_i^{C_i} \text{III} \bullet^{D_i-C_i} \right) \bullet^{T_i-D_i} \right)^*$ . Reciprocally, by definition, every word of this center language is the past of a valid processor allocation configuration.

**Definition 4** *We call Time-Valid Behaviour of  $\tau_i$  every word of*

$$\text{Center} \left( \bullet^{r_i} \left( \left( a_i^{C_i} \text{III} \bullet^{D_i-C_i} \right) \bullet^{T_i-D_i} \right)^* \right)$$

A time-valid behaviour of  $\tau_i$  models a processor time allocation (for  $\tau_i$ ) such that we can guarantee that, in the future, the past of  $\tau_i$  can effectively be extended according to  $\tau_i$  time constraints. In the following, we note  $\mathcal{L}(\tau_i)$  this language.

**Remark 1** *By following a similar line of reasoning, we can show that the time-valid behaviours of a sporadic job  $\tau_i$ , of time parameters<sup>2</sup>  $C_i$ ,  $D_i$  and  $T_i$  belong to*

$$\text{Center} \left( \bullet^* \left( \left( a_i^{C_i} \text{III} \bullet^{D_i-C_i} \right) \bullet^{T_i-D_i} \bullet^* \right)^* \right)$$

In the following, all the techniques we use and the properties we obtain arise from the property  $\mathcal{L}(\tau_i)$  collects all valid behaviours. Then, since this property also applies to sporadic tasks, all these techniques and properties may be applied to periodic as well as sporadic jobs. For simplicity's sake, in the following, we deal only with periodic jobs, but all the obtained results are applicable for the general case.

### 3.3 Model for concurrency: the language of system-valid behaviours

In order to model concurrency, we use the homogeneous product of regular languages, which is defined as follows:

**Definition 5** *Let  $\Sigma_1$  and  $\Sigma_2$  be finite alphabets, and  $L_1 \subset \Sigma_1^*$  and  $L_2 \subset \Sigma_2^*$ .*

- *Let  $\alpha = \alpha_1\alpha_2\ldots\alpha_n \in L_1$  and  $\beta = \beta_1\beta_2\ldots\beta_n \in L_2$  two words of the same length  $n$ . The homogeneous product of  $\alpha$  and  $\beta$  is the word  $\alpha\Omega\beta = \begin{pmatrix} \alpha_1 \\ \beta_1 \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} \cdots \begin{pmatrix} \alpha_{|\alpha|} \\ \beta_{|\beta|} \end{pmatrix} \in (\Sigma_1 \times \Sigma_2)^n$ .*

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<sup>2</sup> recall that for sporadic jobs,  $T_i$  specifies the minimum time interval between two successive instances of the alarm signal associated with  $\tau_i$ .

- The homogeneous product of languages  $L_1$  and  $L_2$  is the language  $L_1\Omega L_2 = \bigcup_{n \in \mathbb{N}} \left( \bigcup_{\substack{\alpha \in L_1 \cap \Sigma_1^n \\ \beta \in L_2 \cap \Sigma_2^n}} \{\alpha\Omega\beta\} \right)$

$\Omega$  is a binary operator. Then, the expression  $(a\Omega b)\Omega c$  is equal to  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ , and the expression  $a\Omega(b\Omega c)$  is equal to  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ . In this case, the semantics associated with a vector is *simultaneity*. Then,  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$  and  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$  share the same semantics, which is *simultaneous execution of  $a$ ,  $b$  and  $c$* . Of course, such semantics can also be associated with the vector  $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ . We consider that  $\begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} a \\ b \\ c \end{pmatrix} \equiv \begin{pmatrix} a \\ b \\ c \end{pmatrix}$ . It follows that the operator  $\Omega$  is associative, and can naturally be generalized to  $n$ -uples of languages.

A behaviour of  $\Gamma$  corresponds to a  $n$ -uple of behaviours of the  $\tau_i$ 's, the semantics associated with  $\begin{pmatrix} a \\ b \end{pmatrix}$  being simultaneity of  $a$  and  $b$ . That is why the set of behaviours of  $\Gamma$  is  $\bigcup_{i=1}^{i=n} \Omega(\mathcal{L}(\tau_i))$ . The homogeneous product does not reduce the set of behaviours for jobs. And since all  $\mathcal{L}(\tau_i)$ 's are centers of regular languages,  $\bigcup_{i=1}^{i=n} \Omega(\mathcal{L}(\tau_i))$  is also a center of language.

If there are fewer processors than jobs (a frequent case!), owning a processor is a resource-sharing problem. It is integrated within the model thanks to Arnold-Nivat's technique [6]: for a  $p$  processor architecture, we take into account the language

$$S_p = \left\{ \omega \in \prod_{i=1}^{i=n} \{\bullet, a_i\} \text{ such that } |\omega|_{\bullet} \geq n - p \right\}$$

that collects the instantaneous configurations corresponding to valid executions on  $p$  processors. The language  $\bigcup_{i=1}^{i=n} \Omega(\mathcal{L}(\tau_i)) \cap S_p^*$  collects all time behaviours that are compatible with both  $\tau_i$ 's time specifications and  $p$ -processor hardware architectures.

The class of regular language centers is not closed by intersection. Here, this property means that processor-sharing can lead a real-time job system to miss at least one of the imposed deadlines. Since we are interested in the set of time-valid behaviours, we only consider the subset of  $\bigcup_{i=1}^{i=n} \Omega(\mathcal{L}(\tau_i)) \cap S_p^*$  that collects the time-valid behaviours (i.e. the scheduling sequences that can be indefinitely extended according to the system time constraints) for the whole real-time system.

The language  $\bigcap_{i=1}^{i=n} (\mathcal{L}(\tau_i)) \cap S_p^*$  is partitioned into two languages

$$L_{inf} = Center \left( \bigcap_{i=1}^{i=n} (\mathcal{L}(\tau_i)) \cap S_p^* \right)$$

and

$$L_{fin} = \left( \bigcap_{i=1}^{i=n} (\mathcal{L}(\tau_i)) \cap S_p^* \right) \setminus Center \left( \bigcap_{i=1}^{i=n} (\mathcal{L}(\tau_i)) \cap S_p^* \right)$$

$L_{inf}$  collects all behaviours compatible with the  $p$ -processor constraint, and which can be infinitely extended according to  $\tau_i$ 's time specifications: they are *valid* behaviours. On the contrary,  $L_{fin}$  collects other behaviours: since they do not belong to the center of the language, they model CPU allocation sequences ensuring that, in the future, at least one of the  $\tau_i$ 's will miss its deadline. Then,  $L_{inf}$  indicates the exact set of time-valid behaviours.

This is why we define the set of time-valid behaviours of a job system in the following way:

**Definition 6** *We call time-valid behaviour for the system  $\Gamma$  within the context  $ICCT(p)$  every element of the language*

$$\mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right) = Center \left( \bigcap_{i=1}^{i=n} (\mathcal{L}(\tau_i)) \cap S_p^* \right)$$

### 3.4 Feasibility Decision

The language  $\mathcal{L}(\tau_i)$  is implemented through its canonical associated finite automata  $A(\tau_i)$  (they follow generic structures: see Figure 2). Thus, the language  $\mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right)$  is also implemented thanks to the computing of its associated finite automaton  $A \left( \frac{\Gamma}{ICCT(p)} \right)$ , which is calculated from the  $A(\tau_i)$ 's through algebraic operations on automata. The decisions pertaining to these languages arise from the structural properties of the corresponding automata. Thus, the existence of these automata is necessary to reach the decision. That is why, in the following, we always show that synchronized products are regular.

The language  $\mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right)$  is designed to contain all valid behaviours of the system  $\Gamma$ . Thus, deciding on the feasibility of  $\Gamma$  under the context  $ICCT(p)$  comes about in an obvious way through the construction of this language:

**Theorem 1** *The system  $\Gamma$  is valid within the context  $ICCT(p)$  if and only if  $\mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right) \neq \emptyset$*

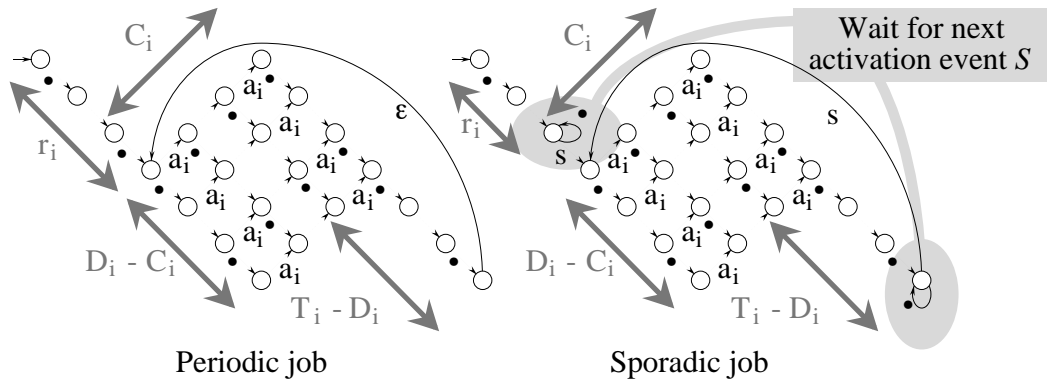
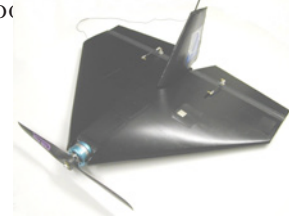


Figure 2. Generic automata for periodic and sporadic jobs

### 3.5 A case study: the AMADO project



From 2002 through July 2005, the French National Research Agency for Space and Aeronautics (ONERA) organized an international competition for the design of miniature unmanned air vehicles (UAV): the AMADO<sup>3</sup> project (see [1] for details). This competition was open to all universities and engineering schools in the world, it was sponsored by french *Délégation Générale de l'Armement* (DGA), of the french defense ministry.

One goal of this competition was to demonstrate the technical feasibility of miniature UAV as an infantry aid. Even though DGA is an organisation devoted to weapon conception, UAVs were designed for the sake of observation, not destruction; they may be viewed as flying binoculars.

Three Poitiers-based labs<sup>4</sup> took part in this competition, and are designing a UAV prototype.

In this paper, we are only interested in controller program design, and more specifically in its autonomous running mode. In this context, the embedded software must drive the UAV in autonomous mode. This part of the software is composed of 7 periodic jobs with invariable CPU load. These jobs share 6 critical resources, which are not considered in this section. Figure 3 presents its DARTS preliminary diagram. Time specifications for the different jobs that compose this controller are the following:

<sup>3</sup> Automated Miniature Aircraft for Detection and Observation.

<sup>4</sup> - LISI (Research Ministry Team (EA) nr 1232): Lab of Applied Computer Science  
- SIC (CNRS Emerging Team (FRE) nr 2731): Signal, Image, Communication  
- LEA (CNRS Research Unit (UMR) nr 6609): Lab of Aerodynamics

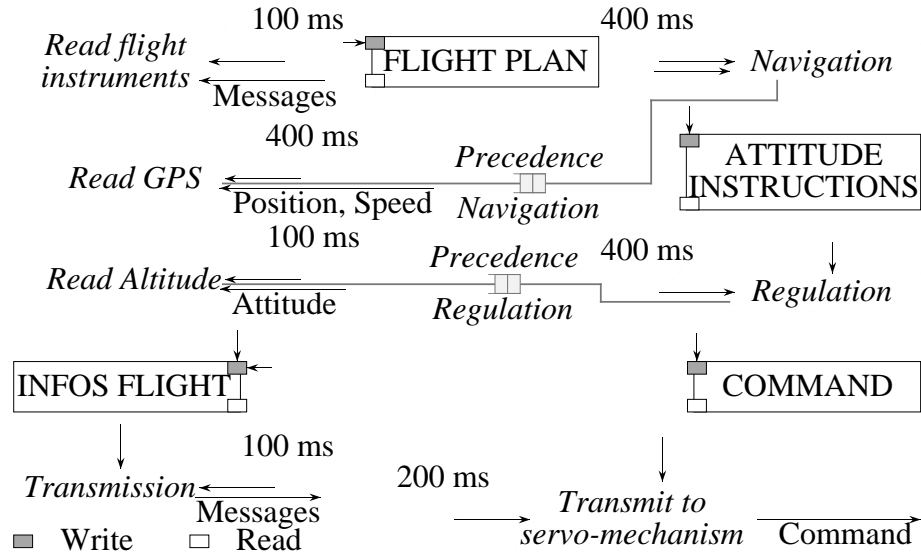


Figure 3. Preliminary DARTS diagram for the UAV controller

| Job      | Role                        | $r_i$ | $D_i$ | $T_i$ | $C_i$ |
|----------|-----------------------------|-------|-------|-------|-------|
| $\tau_1$ | Read Attitude               | 0     | 5     | 5     | 4     |
| $\tau_2$ | Read Flight Instruments     | 0     | 5     | 5     | 4     |
| $\tau_3$ | Read GPS                    | 0     | 12    | 25    | 10    |
| $\tau_4$ | Transmit to servo-mechanism | 0     | 6     | 10    | 2     |
| $\tau_5$ | Transmission                | 0     | 5     | 5     | 2     |
| $\tau_6$ | Navigation                  | 0     | 15    | 25    | 10    |
| $\tau_7$ | Regulation                  | 0     | 16    | 25    | 10    |

Computation of the validation automaton for this system is carried out following the sequence presented below, which shows both the order of the computing and the sizes (edge counting) of homogeneous ( $\Omega$ ) as well as synchronized ( $\Pi$ ) product automata. Recall that homogeneous products model concurrency, and synchronized products model processor sharing. The reader may note that in the following table, both  $|\Omega|$  and  $|\Pi|$  share the same values for subsystems limited to the first four jobs: since we have four processors at our disposal, there is no processor sharing, so synchronized products are not computed. On the contrary, there is processor sharing as soon as the 5<sup>th</sup> job is integrated in the product, and then there exist differences between  $|\Omega|$  and  $|\Pi|$ .

| job      | $ A_i $ | $\frac{ \Omega }{ \Pi }$ | $\frac{ \Omega }{ \Pi }$ | $\frac{ \Omega }{ \Pi }$ | $\frac{ \Omega }{ \Pi }$ | $\frac{ \Omega }{ \Pi }$ | $\frac{ \Omega }{ \Pi }$ |
|----------|---------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\tau_1$ | 13      | $\frac{35}{35}$          | $\frac{591}{591}$        | $\frac{1513}{1513}$      | $\frac{5552}{5544}$      | $\frac{33417}{33041}$    | $\frac{57618}{48975}$    |
| $\tau_2$ | 13      |                          |                          |                          |                          |                          |                          |
| $\tau_3$ | 84      |                          |                          |                          |                          |                          |                          |
| $\tau_4$ | 26      |                          |                          |                          |                          |                          |                          |
| $\tau_5$ | 17      |                          |                          |                          |                          |                          |                          |
| $\tau_6$ | 120     |                          |                          |                          |                          |                          |                          |
| $\tau_7$ | 140     |                          |                          |                          |                          |                          |                          |

The final product is not empty: this system is feasible for a 4-processor target, without considering critical resource sharing.

## 4 Integrating job constraints

In this section, we integrate job interdependence to the model, and we show that the validation results always stand. The constraints we wish to consider are critical resource sharing and message-based communications. Thus, we consider the context

| Hardware     |               |               | Software        |                 |             |
|--------------|---------------|---------------|-----------------|-----------------|-------------|
| Architecture | Clock         | Communication | Structure       | Synchronisation | Preemption  |
| $p \geq 1$   | <i>Common</i> | $(H^0, s)$    | $(p, r_i, C_f)$ | <i>rc</i>       | <i>totp</i> |

Let  $\Delta$  be the set of items (resources, messages, etc.) concerning the job system synchronisation. The corresponding context is called  $D(\Delta)CCT(p)$  (with  $D(\Delta)$  for *dependent jobs under constraint set  $\Delta$* ) in the following.

### 4.1 Using Arnold-Nivat synchronized products

To integrate job interdependence (communication, resource sharing) between the  $\tau_i$ 's, we also use Arnold-Nivat's technique [6]: each resource  $R$  (shared resource or communication message) is modeled by a virtual job  $\xi_R$ , designed to trace its states ( busy/idle, for instance, for a shared resource using a basic protocol, but more highly elaborated protocols can likewise be modeled [7] ).

The principle of the Arnold/Nivat model is presented in Figure 4:

- We consider a set of tasks sharing a critical resource: each task is modeled by a finite automaton. The first step consists in building the product automaton, which models the concurrent system composed of all the tasks.
- The protocol used to manage the constraint (shared resource or any other constraint) is a process: it is modeled by the use of a specific automaton (see [7], for instance). In the case of Figure 4, the constraint consists in executing  $b$  before executing  $\psi$  (precedence constraint). Step 2 consists in producing the product automaton that models the concurrent running of both the task system and the protocol. Step 3 consists in erasing all states (and corresponding edges) of the automaton which correspond to a violation of the protocol.
- The product automaton is composed of the states which satisfy the constraint: the protocol component is now useless (it is not part of the task system), it can be erased through use of a projection (step 4).

The general process can be viewed as follows .

$\left(\bigcup_{i=1}^{i=n} \mathcal{L}(\tau_i)\right) \Omega \mathcal{L}(\xi_R)$  collects the behaviours of the system composed of both the  $\tau_i$ 's and the resource  $R$ . Let us now consider the  $S_R$  set of the instantaneous configurations compatible with the resource protocol management.  $\left(\left(\bigcup_{i=1}^{i=n} \mathcal{L}(\tau_i)\right) \Omega \mathcal{L}(\xi_R)\right) \cap S_R^*$  collects only the behaviours of  $\Gamma$  which are compatible with the resource protocol management.

Now, we use the same property as for processor sharing (see Section 3.4): the class of regular language centers is not closed by intersection. Here, this property means that sharing a critical resource can lead a real-time system to miss at least one of the imposed deadlines. Since we are still interested in the set of time-valid behaviours, we again consider the subset of  $\left(\left(\bigcup_{i=1}^{i=n} \mathcal{L}(\tau_i)\right) \Omega \mathcal{L}(\xi_R)\right) \cap S_R^*$  that collects the time-valid behaviours (i.e. the scheduling sequences that can be indefinitely extended according to the system time constraints) for the whole system. Following the same reasoning as for context  $ICCT(p)$ , we collect all infinitely extendable behaviours of this set, i.e. we consider

$$\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(\infty)}\right) = Center\left(\left(\left(\bigcup_{i=1}^{i=n} \mathcal{L}(\tau_i)\right) \Omega \mathcal{L}(\xi_R)\right) \cap S_R^*\right)$$

This language is associated with the context  $D(\Delta)CCT(\infty)$ , because it collects all time-valid behaviours according to both time and resource protocol constraints, but without considering processor sharing constraints. These can be integrated in the model through renewed use of Arnold-Nivat's technique, in the same way as for  $ICCT(p)$  context, i.e. computing  $\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(p)}\right) = Center\left(\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(\infty)}\right) \cap S_p^*\right)$ .



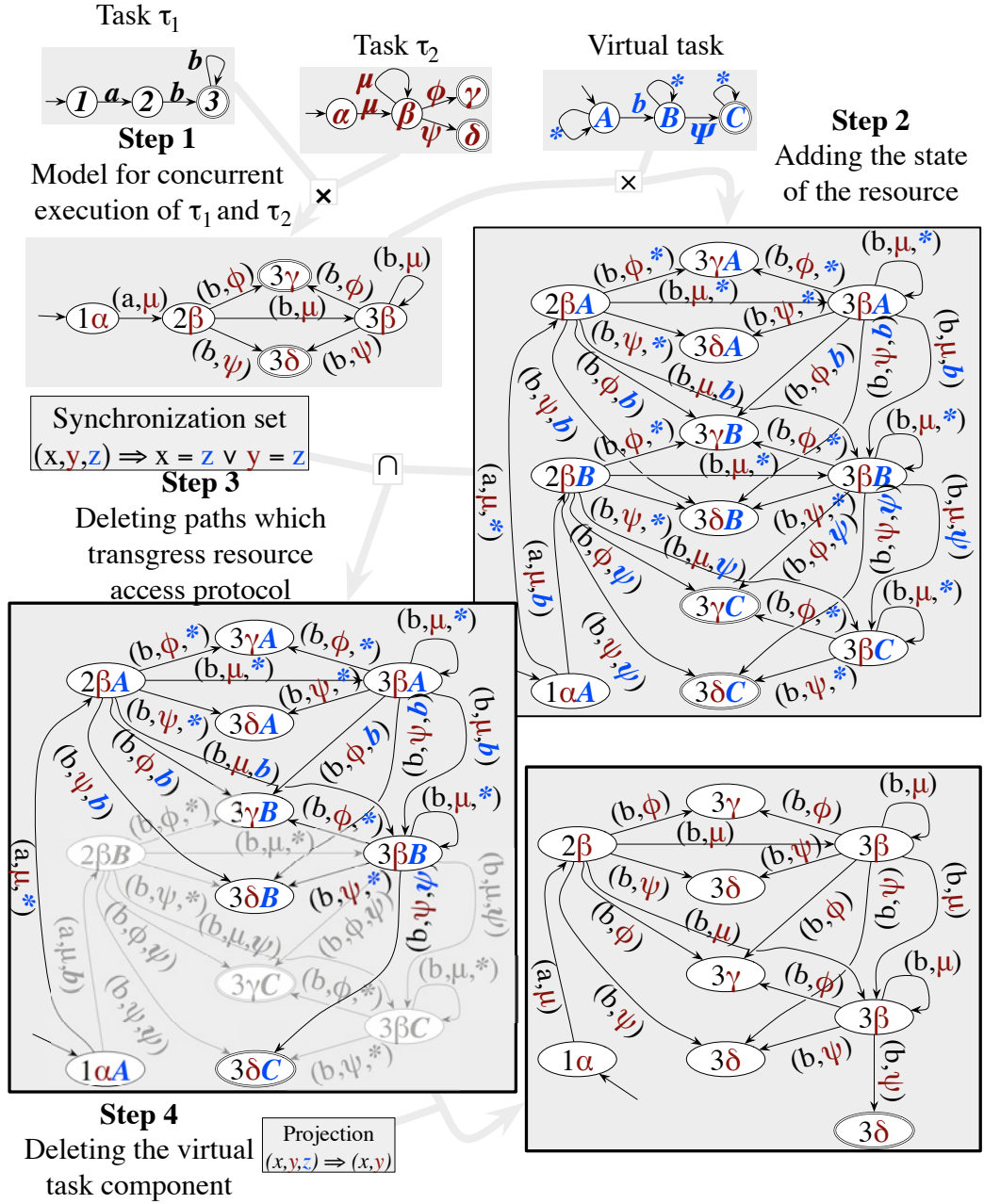


Figure 4. Arnold-Nivat model for resource sharing

**Remark 2** Let  $\rho_j$  be the projection  $(x_i)_{i \in [1, n]} \rightarrow x_j$ , and let us extend this notation to intervals, by defining  $\rho_{[a, b]}$  as the projection

$$(x_i)_{i \in [1, n]} \rightarrow (x_i)_{i \in [\text{Max}(1, a), \text{Min}(b, n)]}$$

In the language  $\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(p)}\right)$ , the components corresponding to virtual jobs modeling resources are now useless: we erase them from the model, by using  $\rho_{[1, n]}$ . Thus, we finally consider the language  $\rho_{[1, n]}\left(\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(p)}\right)\right)$ . For the sake of simplicity, we shall denote this language by  $\mathcal{L}\left(\frac{\Gamma}{D(R)CCT(p)}\right)$ : then, in

the following, we omit writing the  $\rho_i$ 's.

Since a synchronisation model is based upon intersections of languages, we obtain

$$\mathcal{L} \left( \frac{\Gamma}{D(\{R_1, R_2\}) CCT(p)} \right) = Center \left( \mathcal{L} \left( \frac{\Gamma}{D(R_1) CCT(p)} \right) \cap S_{R_2}^* \right)$$

Applying this property by induction, we obtain the following definition:

**Definition 7** Let  $\Gamma$  be a real-time system and  $\Delta = (R_j)_{j \in [1, r]}$  be a set of synchronization constraints (resource or messages). We call time-valid behaviour of  $\Gamma$  under the context  $D(\Delta) CCT(p)$  every element of the language

$$\mathcal{L} \left( \frac{\Gamma}{D(\Delta) CCT(p)} \right) = Center \left( \mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right) \cap \left( \bigcap_{i=1}^{i=n} (S_{R_i}^*) \right) \right)$$

where  $S_{R_i}$  is the set of valid instantaneous configurations, according to resource  $R_i$  management protocol.

In [6], the author shows how this technique can be used in order to model resource sharing, precedence and communication by message.

#### 4.2 Feasibility Decision

Like  $\mathcal{L} \left( \frac{\Gamma}{ICCT(p)} \right)$ ,  $\mathcal{L} \left( \frac{\Gamma}{D(\Delta) CCT(p)} \right)$  is a language that collects time-valid behaviours of the system. Then, theorem 1 remains valid. Thus, we obtain:  $\Gamma$  is valid under context  $D(\Delta) CCT(p) \Leftrightarrow \mathcal{L} \left( \frac{\Gamma}{D(\Delta) CCT(p)} \right) \neq \emptyset$ .

#### 4.3 The AMADO project: feasibility with critical resources

Let us now complete the feasibility study we began in Section 3.5, by including critical resource sharing in the synchronized product computing. First, we complete the table that presents time characteristics of jobs along with the resources used (resources are identified as  $R_1$ ,  $R_2$ , and so on):

| Job      | Role                        | $r_i$ | $D_i$ | $T_i$ | $C_i$ | Resources used  |
|----------|-----------------------------|-------|-------|-------|-------|---|
| $\tau_1$ | Read Attitude               | 0     | 5     | 5     | 4     | Infos Flight ( $R_1$ )<br>Precedence Regulation ( $R_2$ )                                   |
| $\tau_2$ | Read Flight Instruments     | 0     | 5     | 5     | 4     | Flight plan ( $R_3$ )<br>Command ( $R_4$ )  |
| $\tau_3$ | Read GPS                    | 0     | 12    | 25    | 10    | Infos Flight ( $R_1$ )<br>Precedence Navigation ( $R_5$ )                                   |
| $\tau_4$ | Transmit to servo-mechanism | 0     | 6     | 10    | 2     | Command ( $R_4$ )   |
| $\tau_5$ | Transmission                | 0     | 5     | 5     | 2     | Infos Flight ( $R_1$ )  |
| $\tau_6$ | Navigation                  | 0     | 15    | 25    | 10    | Precedence Navigation ( $R_5$ )<br>Flight plan ( $R_3$ )<br>Attitude Instructions ( $R_6$ ) |
| $\tau_7$ | Regulation                  | 0     | 16    | 25    | 10    | Precedence Regulation ( $R_2$ )<br>Command ( $R_4$ )<br>Attitude Instructions ( $R_6$ )     |

Computing the validation automaton for this system is carried out following the sequence presented on the table drawn below, in the same way as for an independent job case:  $\Omega$  and  $R_j$   $\Pi$  denote homogeneous and resource  $R_j$  synchronized product automata, and  $C$  denotes the center language automaton, which is computed after each synchronized product. Each synchronization addresses both resource and processor sharing. The reader can note that in the following computation, tasks are integrated within the product following a different order than in § 3.5: we begin by integrating the more constrained tasks, in order to limit product automata sizes as much as possible.

|          | $ A_i $ | $\frac{ \Omega }{R_j\left(\frac{ \Pi }{ C }\right)}$ |  |   |   |  |   |
|----------|---------|--|--|---|---|--|---|
| $\tau_7$ | 140     | 1055   | $\frac{8233}{R_5\left(\frac{8179}{8179}\right)}$ | $\frac{20868}{R_3\left(\frac{18429}{17358}\right)}$ | $\frac{23098}{R_2\left(\frac{17710}{15701}\right)}$ | $\frac{15701}{R_1\left(\frac{3939}{0}\right)}$ | $\frac{0}{R_4\left(\frac{0}{0}\right)}$ |
| $\tau_6$ | 120     | $R_6\left(\frac{884}{786}\right)$                    |  | $R_4\left(\frac{14083}{10210}\right)$               | $R_1\left(\frac{11506}{9946}\right)$                |  |   |
| $\tau_3$ | 84      |  |  |   |   |  |   |
| $\tau_2$ | 13      |  |  |   |   |  |   |
| $\tau_1$ | 13      |  |  |   |   |  |   |
| $\tau_5$ | 17      |  |  |   |   |  |   |
| $\tau_4$ | 26      |  |  |   |   |  |   |

The final product is empty: this system is not feasible for a 4-processor target, according to critical resource sharing. Since this system is feasible without considering resource sharing (see §3.5), the resources are responsible for unfeasibility.

Let us now observe the synchronization constraint levels involved by each resource, for instance by evaluating the ratios  $\frac{|C|}{|\Pi|}$ . With this objective, for each resource  $R_i$ , we study the subsystem composed of all jobs that share  $R_i$ , without processor sharing (i.e. using as many processors as jobs). Resource  $R_1$  is the most constraining: it leads, for a 3-job system, to a level of  $\frac{94}{654}$ , that is to say about 16%. Overloading  $R_1$ -sharing with processor sharing, this level drops to 9% for a 2-processor target, and to 1% for a 1-processor. That is why the whole system is not feasible. The feasibility system diagnosis advises the conceptor to observe  $R_1$  sharing to render the system feasible.

## 5 WCET free analysis

In general, real-time jobs execute programs whose CPU load is variable: they contain both *if...then...else* or *for...do* statements. In this case, the CPU load of each instance of the job depends on the values of some of its variables, i.e. its functional semantics. In previous sections, the task  $\tau_i$  is associated with the time parameter  $C_i$ , which specifies its CPU allocation time. This value must be viewed as the WCET: it is the minimal CPU allocation time needed to schedule  $\tau_i$  in the worst case (see Figure 6).

In this section, we are extending our model in order to analyse systems in accordance with the real values of  $C_i$ , as opposed to WCET values. We have termed this manner of proceeding WCET-free analysis.

In this study, we consider programs composed of both atomic ( $:=$ ), choices (*if...statements*) and static loop statements (Ada like *for...*). We consider neither dynamic loops nor recursive subroutine calls: in the context of hard real-time software, this restriction is realistic.

Thus, the context considered here is

| Hardware     |               |               | Software        |                 |             |
|--------------|---------------|---------------|-----------------|-----------------|-------------|
| Architecture | Clock         | Communication | Structure       | Synchronisation | Preemption  |
| $p \geq 1$   | <i>Common</i> | $(H^0, s)$    | $(p, r_i, C_v)$ | <i>rc</i>       | <i>totp</i> |

This context is called  $D(\Delta)CCVT(p)$  (with  $V$  for *variable CPU load tasks*) in the following.

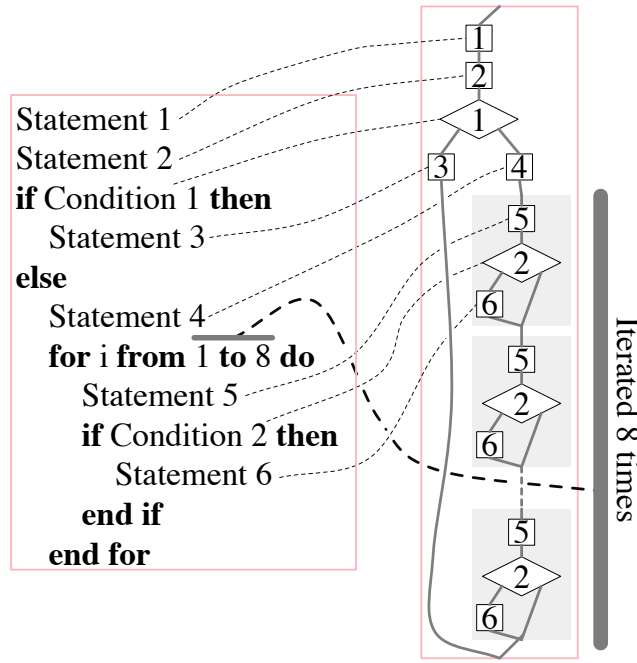


Figure 5. From the program to the graph

### 5.1 The language of job-valid behaviours

To integrate these kinds of jobs, one must model many CPU loads for each  $\tau_i$ . However, not all CPU loads are allowed: the only ones allowed are those which effectively correspond to at least one of the functional behaviours of the job. Let us consider, for instance, the program presented on Figure 5. It is associated with its canonical finite automaton representation by applying the morphism  $Statement \rightarrow a_i$  to all values that label edges (they are statements). Thus, every path through this automaton is labelled with a word  $a_i^j$ , where  $j$  is a CPU allocation duration compatible with at least one of the behaviours of  $\tau_i$ . In this example, the set is  $a^2 \{a^2, a^2 (a^2 \{a, a^2\})\}$ , which can also be described by  $\{a^4, (a^{28+i})_{i \in [1,8]}\}$ . We denote this set with  $P_{\tau_i}$ . It is always finite, because there are no dynamic statements in the program (no dynamic loops, no recursive subroutine calls). For  $\omega \in P_{\tau_i}$ ,  $Center \left( \bullet^{r_i} \left( \left( \omega \text{III} \bullet^{D_i - |\omega|} \right) \bullet^{T_i - D_i} \right)^* \right)$  collects the set of time-valid behaviours of  $\tau_i$  such that each instance of  $\tau_i$  uses a CPU allocation of  $|\omega|$  time units. Now, let us call  $(\omega_k)_{k \in I}$  the words that belong to  $P_{\tau_i}$ . Since  $P_{\tau_i}$  is finite,  $I$  is finite too. In general, time-valid behaviours of successive instances of  $\tau_i$  correspond to different  $\omega_k$ 's. Then, a time-valid behaviour of  $\tau_i$  belongs to  $\mathcal{L}(\tau_i) = Center \left( \bullet^{r_i} \left( \left( \bigcup_{k \in I} \left( \omega_k \text{III} \bullet^{D_i - |\omega_k|} \right) \right) \bullet^{T_i - D_i} \right)^* \right)$ . Since  $I$  is a finite set, this language remains regular.

The  $\mathcal{L}(\tau_i)$ 's being sets of  $\tau_i$ 's valid behaviours, computing the set of valid behaviours of the whole system  $\Gamma$  is based upon both homogeneous products

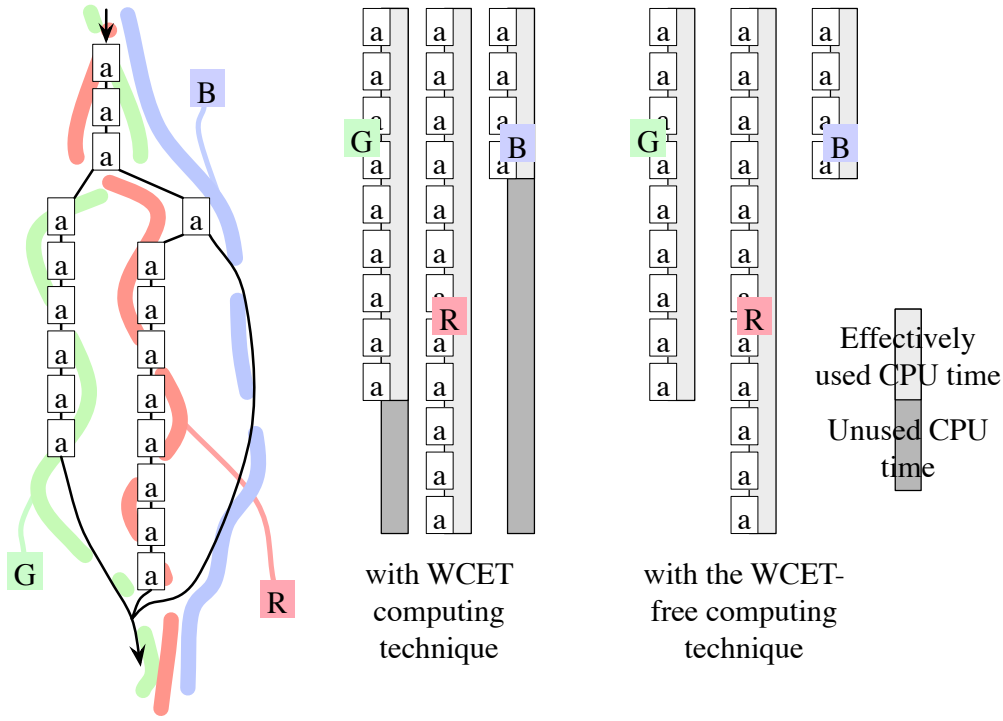


Figure 6. From the graph to the CPU load set

and Arnold-Nivat's synchronisation technique.

## 5.2 Feasibility Decision

The role of Arnold-Nivat's synchronization technique is to forbid some behaviours of the  $\tau_i$ 's. Under the context  $D(\Delta) CCT(p)$ , forbidding a behaviour corresponds to forbidding a specific CPU allocation sequence, but no restrictions are imposed on specific functional behaviours of the job.

Under the context  $D(\Delta) CCVT(p)$ , synchronizing may forbid  $\tau_i$  from following some of its behaviours (the shorter ones, for instance), and allow it to follow others (longer...). Thus, the predicate  $\mathcal{L}\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right) \neq \emptyset$  reaches the property *for each job, and there exists at least one valid path*. We call this property *weak feasibility*:

**Notation 1** Recall that we are noting  $\rho_i$  the projection of a vector  $V$  on its  $i^{th}$  component. At this point, we are introducing a new projection:  $\pi$ .

Let  $A$  be an alphabet, and  $B \subset A$ . We note respectively  $\pi_B$  and  $\pi_{-B}$  the two morphisms

$$\pi_B : A \rightarrow B, \begin{cases} x \in B \rightarrow x \\ x \in A \setminus B \rightarrow \epsilon \end{cases} \quad \text{and} \quad \pi_{\neg B} : A \rightarrow B, \begin{cases} x \in B \rightarrow \epsilon \\ x \in A \setminus B \rightarrow x \end{cases}$$

When  $A = \{a\}$ , we note  $\pi_{\{a\}}$  as well as  $\pi_a$ .

**Definition 8** A real-time system  $\Gamma = \{(\tau_i)_{i \in [1, n]}\}$  is weakly feasible under context  $C$  if  $\forall i \in [1, n], \exists \omega \in P_{\tau_i}$  such that  $\exists \alpha \in \mathcal{L}\left(\frac{\Gamma}{C}\right)$  such that  $\pi_{a_i}(\rho_i(\alpha)) = \omega$

Thus, theorem 1 addresses weak feasibility.

When job CPU loads are invariable, *weak feasibility* and *feasibility* are equivalent, since the only two alternatives are the existence and the non-existence of one occurrence of  $\omega$ . That is why theorem 1 is useful when addressing feasibility in this context.

On the other hand, as soon as there exists one  $\tau_i$  whose successive instances follow different CPU loads, this theorem is useless, since *weak feasibility* and *feasibility* are no longer equivalent: for such systems, weak feasibility means that there are compatible behaviours. Others can nevertheless be forbidden, because of functional incompatibilities (resource sharing, for instance).

Let us consider, for instance, a job system  $(\tau_i)_{i \in [1, 2]}$ , where  $\tau_1$  and  $\tau_2$  share a critical resource. The body of  $\tau_1$  is presented in Figure 7.a. The structures of their respective bodies are such that, when resource sharing is used, both of the two jobs miss their deadlines. However, for each of these two jobs, there exists a possible behaviour compatible with its time constraints: for  $\tau_1$ , it is the set of behaviours which avoids resource sharing (see Figure 7.b). Thus, the system is not feasible since some functional behaviours of jobs are excluded from systemic constraints, but it is weakly feasible: it satisfies theorem 1!

Thus, theorem 1 does not stand within context  $D(\Delta)CCVT(p)$ .

In a feasible system, every job may always be allowed to follow any of its (functional) behaviours according to both resource sharing and time constraints. This property is stronger than weak feasibility. We call it *strong feasibility*:

**Definition 9** A real-time system  $\Gamma = \{(\tau_i)_{i \in [1, n]}\}$  is strongly feasible under context  $C$  if and only if

$$\forall (\omega_i)_{i \in [1, n]} \in \prod_{i=1}^{i=n} P_{\tau_i}, \exists \alpha \in \mathcal{L}\left(\frac{\Gamma}{C}\right) \text{ such that } \forall i \in [1, n], \pi_{a_i}(\pi_i(\alpha)) = \omega_i$$

The feasibility of a system  $\Gamma$  is reached as soon as  $\mathcal{L}\left(\frac{\Gamma}{C}\right) \neq \emptyset$  and  $\Gamma$  is strongly

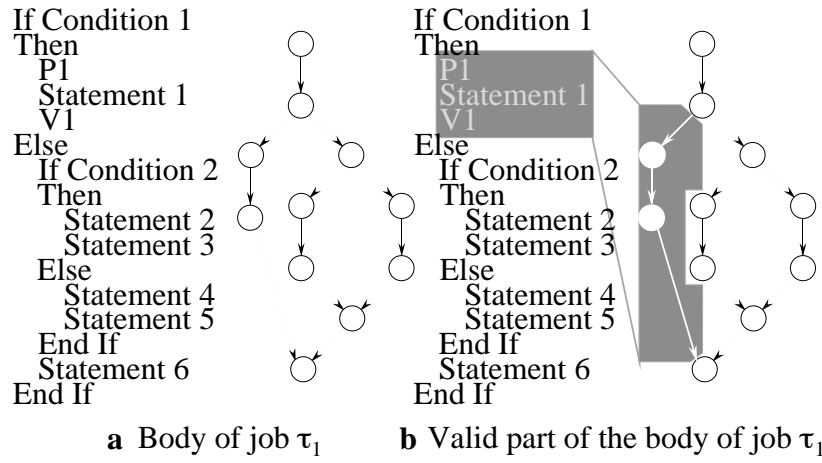


Figure 7. Example of invalid configuration

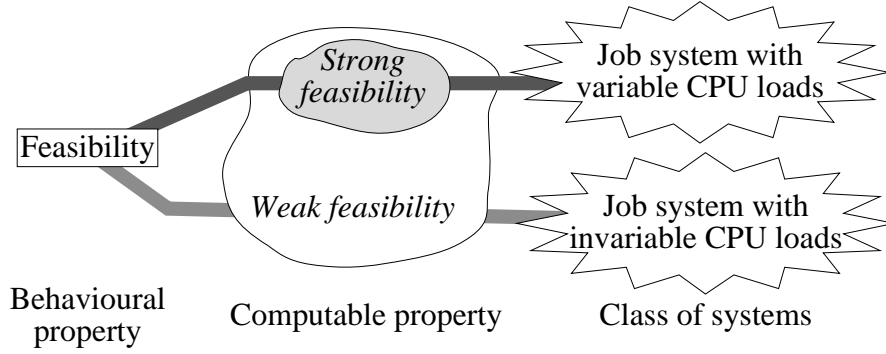


Figure 8. Feasibility computation

feasible. These considerations lead us to generalize theorem 1 in the following way (see Figure 8)

**Theorem 2** A system  $\Gamma = \{(\tau_i)\}_{i \in [1, n]}$  is feasible under context  $D(\Delta)CC$

$$VT(p) \text{ if and only if } \begin{cases} \mathcal{L}\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right) \neq \emptyset \\ \Gamma \text{ is strongly feasible} \end{cases}$$

Note that for invariable CPU load, both weak and strong feasibility are equivalent, and that theorems 1 and 2 address the same property: feasibility.

If  $\Gamma$  satisfies theorem 2, the set  $\mathcal{L}\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$  of its time-valid behaviours is a regular language. For such languages, the star lemma [16] shows that every sufficiently long word contains an iterated subword.

Thus, this lemma leads to the following property: if  $\Gamma$  is a set of interdependent tasks, to be scheduled under the context  $D(\Delta)CCVT(p)$ , the time-valid behaviours of  $\Gamma$  are cyclic.

This consequence shows that the cyclicity problem for multiprocessor contexts



[12] can be solved, hence on-line scheduling can be validated by simulation for multi-processors.

### 5.3 The AMADO project: considering variable CPU load jobs

In the UAV controller, *Navigation* is the most complex job, in terms of control statements. Its program contains some **if/then/else** constructors, and calls for some elementary computing subroutines (3 or 4 atomic statements for each one, without control statements). This program and its translation into an operational model are presented in the table presented in Figure 9.

Translated into a set of operational behaviours, the program presented on this Figure leads to

$$a^3 P(R_5) a^{13} V(R_5) \{a, a^2\} P(R_3) a^{12} V(R_3) \{a, a^2\} P(R_6) a^8 \{a, a^2\}^6 V(R_6) a$$

Since  $\{a, a^2\}^6 = \{a^6, a^7, a^8, a^9, a^{10}, a^{11}, a^{12}\}$ , operational behaviours of this program correspond to 24 different CPU loads, the least loaded being a 51-time-units behaviour, and the most loaded a 59-time-units.

Computing of the synchronized product automaton associated with this variable CPU time real-time system follows this sequence:

|          | $ A_i $ | $\frac{ \Omega }{R_j\left(\frac{ \Pi }{ C }\right)}$ |   |   |   |   |   |                                       |
|----------|---------|--|---|---|---|---|---|---------------------------------------|
| $\tau_7$ | 140     | 2080   | $\frac{13768}{R_5\left(\frac{12866}{12678}\right)}$ | $\frac{32705}{R_3\left(\frac{29382}{27858}\right)}$ | $\frac{30328}{R_2\left(\frac{21269}{18770}\right)}$ | $\frac{21445}{R_1\left(\frac{2160}{1707}\right)}$ | $\frac{2302}{R_4\left(\frac{1324}{379}\right)}$ |                                       |
| $\tau_6$ | 212     | $R_6\left(\frac{2074}{2074}\right)$                  |   | $R_4\left(\frac{25327}{14296}\right)$               |   |   |   | $R_1\left(\frac{15431}{13904}\right)$ |
| $\tau_3$ | 84      |  |   |   |   |   |   |                                       |
| $\tau_2$ | 13      |  |   |   |   |   |   |                                       |
| $\tau_1$ | 13      |  |   |   |   |   |   |                                       |
| $\tau_5$ | 17      |  |   |   |   |   |   |                                       |
| $\tau_4$ | 26      |  |   |   |   |   |   |                                       |

The final product is not empty: in accordance with critical resource sharing, the system is weakly feasible for a 4-processor target, but not strongly feasible: the one and only path accepted for scheduling purposes is the shortest (51 time units).

| Navigation job Program Code  | Operationals  |
|--|---|
| <pre> void Navigation (Job_Traiter_GPS) {   double Pos[3],ActDir,test1;   double NxtPos[3];   double Dir,DifDir;   GetResource(Res_InfosGPS);   Pos[Lat]=LatLngChk(GPS_Tr.Lat,GPS_Tr.LatFrac);   Pos[Lng]=LatLngChk(GPS_Tr.Lng,GPS_Tr.LngFrac);   Pos[FLlev]=GPS_Tr.AltRef/1000.0;   ActDir=GPS_Tr.COG/100;   ReleaseResource(Res_InfosGPS);   if   (     (Abs(NxtPos[Lat],Pos[Lat])&lt;AprxLat)     &amp;&amp;     (Abs(NxtPos[Lng],Pos[Lng])&lt;AprxLng)   )     PntToRchNW=(PntToRchNW+1)%NWPtTotNb;   GetResource(Res_Waypoints);   NxtPos[Lat]=LatCmp(TabWayPoint[PntToRchNW]);   NxtPos[Lng]=LngCmp(TabWayPoint[PntToRchNW]);   NxtPos[FLlev]=FLCmp(TabWayPoint[PntToRchNW]);   ReleaseResource(Res_Waypoints);   if(ActDir&gt;180)     ActDir-=360;   GetResource(Res_ConsignesAttitude);   Dir=DirDet(Pos, NxtPos);   DifDir= Dir- ActDir;   Consigne.WO_Ref=NxtPos[AltCmp]-Pos[AltCmp];   if(Consigne.WO_Ref&gt;VrtLmtSpeed)     Consigne.WO_Ref=VrtLmtSpeed;   if(Consigne.WO_Ref&lt;=-VrtLmtSpeed)     Consigne.WO_Ref=-VrtLmtSpeed;   if(DifDir&gt;180) DifDir-=360;   if(DifDir&lt;-180) DifDir+=360;   Consigne.Phi_Ref=0.2*DifDir;   if(Consigne.Phi_Ref&gt;LmtIncl)     Consigne.Phi_Ref=LmtIncl;   if(Consigne.Phi_Ref&lt;=-LmtIncl)     Consigne.Phi_Ref=-LmtIncl;   ReleaseResource(Res_ConsignesAttitude);   TerminateTask(); } </pre> | <pre> void Navigation () {   a;   a;   a;   P(R<sub>5</sub>);   a<sup>4</sup>;   a<sup>4</sup>;   a<sup>4</sup>;   a;   V(R<sub>5</sub>);   {a, a<sup>2</sup>};   P(R<sub>3</sub>);   a<sup>4</sup>;   a<sup>4</sup>;   a<sup>4</sup>;   V(R<sub>3</sub>);   {a, a<sup>2</sup>};   P(R<sub>6</sub>);   a<sup>4</sup>;   a;   a<sup>3</sup>;   {a, a<sup>2</sup>};   {a, a<sup>2</sup>};   {a, a<sup>2</sup>};   {a, a<sup>2</sup>};   {a, a<sup>2</sup>};   {a, a<sup>2</sup>};   V(R<sub>6</sub>);   a; } </pre> |

Figure 9. Navigation job program vs operationals for one procedure of the AMADO project code

## 6 Integrating hardware specifications

Let us now suppose that  $\Gamma$  is designed to run on a multiprocessor machine where some processors do not run at the same speed. Moreover, we suppose that each  $\tau_i$  is designed to run on a fixed processor (there is no task migration, this hypothesis is realistic in the framework of real-time softwares). Let us consider the context

| Hardware     |                      |               | Software        |                 |             |
|--------------|----------------------|---------------|-----------------|-----------------|-------------|
| Architecture | Clock                | Communication | Structure       | Synchronisation | Preemption  |
| $p \geq 1$   | <i>Common starts</i> | $(H^0, s)$    | $(p, r_i, C_v)$ | <i>rc</i>       | <i>totp</i> |

This context is called  $D(\Delta)SCVT(p)$  (with  $S$  for *Common starting clocks*) in the following.

Let  $\tau_1$  and  $\tau_2$  be two tasks designed to run on processors  $p_1$  and  $p_2$ , whose running speeds are different, i.e. whose CPU time unit are different. We note by  $u_i$  the execution time of any atomic statement on  $p_i$  (for  $i \in \{1, 2\}$ ). Obviously, we suppose that  $u_1 \neq u_2$ . Then, any symbol which appears in words of  $\mathcal{L}(\tau_1)$  (resp.  $\mathcal{L}(\tau_2)$ ) is supposed to be associated with the delay  $u_1$  (resp  $u_2$ ): the time semantics correspondingly associated with the language depends on the target processor. We take this into account by indexing the language with this time unit: the language which collects the behaviours of  $\tau_1$  when it is running on  $p_1$  is denoted  $\mathcal{L}_{u_1}(\tau_1)$ .

In previous sections, all processors were running at the same speed  $u$ , this speed was omitted in the description of the relevant languages, and the synchronized product was naturally associated with  $u$ . At present, however, we cannot compute the synchronized product of  $\mathcal{L}_{u_1}(\tau_1)$  and  $\mathcal{L}_{u_2}(\tau_2)$ ; this is due to the fact that it is impossible to associate a time unit with the edges of the product.

In order to avoid this problem, in this section we are proposing a technique allowing us to determine a unit  $u$  such that

- $\mathcal{L}_{u_1}(\tau_1)$  can be mapped into  $\mathcal{L}_u(\tau_1)$ ,
- $\mathcal{L}_{u_2}(\tau_2)$  can be mapped into  $\mathcal{L}_u(\tau_2)$ ,

These two languages bring us back to the previous context: the technique presented earlier in this article can be used to compute the synchronized product of  $\mathcal{L}_u(\tau_1)$  and  $\mathcal{L}_u(\tau_2)$ , whose edges are naturally associated with the time  $u$ . In Section 6.1, we show how  $u$  can be determined, and we give the mapping to compute  $\mathcal{L}_u(\tau_1)$  from  $\mathcal{L}_{u_1}(\tau_1)$ .

### 6.1 The language of job-valid behaviours

$\mathcal{L}_{u_1}(\tau_1)$  is computed from the time characteristics of  $\tau_1$ . Some of these characteristics are time-absolute values which do not depend on the characteristics of the target processor:  $T_i$ ,  $D_i$  and  $r_i$ . On the other hand, the value  $C_i$  depends

on the speed of the processor: we get

$$\frac{C_i}{CPU \text{ with a time unit } u} = \frac{\frac{C_i}{k}}{CPU \text{ with a time unit } \frac{u}{k}}$$

Computing  $\mathcal{L}_u(\tau_1)$  from  $\mathcal{L}_{u_1}(\tau_1)$  presupposes our keeping the specified values of  $T_i$ ,  $D_i$  and  $r_i$ , and likewise presupposes our considering for  $C_i$  the value corresponding to the time unit  $u$ .

**Example 1** Let  $\tau_i$  be a job of characteristics  $r_i = 3ms$ ,  $D_i = 8ms$ ,  $T_i = 10ms$  and  $C_i = 3u_i$ . With  $u_i = 1ms$ , we get

$$\mathcal{L}_{u_i}(\tau_i) = Center \left( \bullet^3 \left( (a^3 \text{III} \bullet^5) \bullet^2 \right)^* \right)$$

With  $u_i = 250 \text{ ns}$ , we get the (different) language

$$\mathcal{L}_{u_i}(\tau) = Center \left( \bullet^{12} \left( (a^3 \text{III} \bullet^{29}) \bullet^8 \right)^* \right)$$

Observing Figure 10, one may note that  $u_1$  and  $u_2$  must be multiples of  $u$ : the natural value for  $u$  is  $\gcd(u_1, u_2)$ , which can always be computed as soon as  $u_1$  and  $u_2$  are integer values. This last property is yielded as soon as time units are expressed in terms compatible with the characteristics of the processors put to work ( $\mu s$ ,  $ns$ ,  $ps$ , etc.).

Words belonging to  $\mathcal{L}_{u_i}(\tau_i)$  integrate parameters  $r_i$ ,  $D_i$  and  $T_i$  by enumerating job inactivity in terms of time units. A word which describes the behaviour of a task on a time interval of duration  $t$  is composed of letters whose individual time interval is of length  $u_i$ , then  $t$  must be a multiple of  $u_i$ :

$$\begin{array}{c} \xrightarrow{t} \\ \xleftrightarrow{u_i} \xleftrightarrow{u_i} \dots \xleftrightarrow{u_i} \\ \xleftrightarrow{a} \xleftrightarrow{a} \dots \xleftrightarrow{a} \end{array} . \text{ Computing the language } \mathcal{L}_{u_i}(\tau_i) \text{ involves the translat-}$$

ing of time values  $T_i$ ,  $D_i$  and  $r_i$  into word lengths. For  $T_i$ , for instance, we get

$$\text{the Figure } \begin{array}{c} \xrightarrow{T_i} \\ \xleftrightarrow{u_i} \xleftrightarrow{u_i} \dots \xleftrightarrow{u_i} \\ \xleftrightarrow{x} \xleftrightarrow{x} \dots \xleftrightarrow{x} \end{array} : \text{ the inactivity of a task for a time } T_i \text{ would}$$

then be expressed by the word  $\bullet^{\frac{T_i}{u_i}}$ .

On the one hand  $r_i$ ,  $D_i$  and  $T_i$  are usually greater than  $20ms$ , on the other hand  $u_i$  is about some  $\mu s$ . Since external time specifications for jobs can often be reviewed by constraining or relaxing them (these modifications must be compatible with external specifications of the captors and activators), we suppose in the following that  $\frac{T_i}{u_i} \in \mathbb{N}$ ,  $\frac{D_i}{u_i} \in \mathbb{N}$  and  $\frac{r_i}{u_i} \in \mathbb{N}$ .

The expression of  $\mathcal{L}_{u_i}(\tau_i)$  comes from the translation into time units of the time specifications of the job (period, deadline and first activation date). Then,

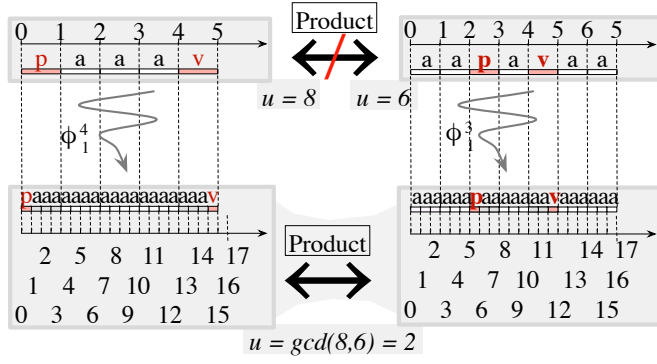


Figure 10. CPU time units and observation time units

for the job  $\tau_i$ , whose behaviours of  $P_{\tau_i}$  are the  $(\omega_k)_{k \in I}$  (see §5.1), we obtain

$$\mathcal{L}(\tau_i) = \text{Center} \left( \bullet \frac{r_{\tau}}{u_i} \left( \left( \bigcup_{k \in I} \left( \omega_k \text{III} \bullet \frac{D_i - |\omega_k|}{u_i} \right) \right) \bullet \frac{T_i - D_i}{u_i} \right)^* \right)$$

We call *Validity class* of  $\tau_i$  the set

$$\mathbb{L}(\tau_i) = \{ \mathcal{L}_u(\tau_i), \text{ where } u \in \mathbb{N}^*, r_i \in u\mathbb{N}, D_i \in u\mathbb{N} \text{ and } T_i \in u\mathbb{N} \}$$

This set collects all languages that model operational behaviors of  $\tau_i$  on all possible classes of targets, in terms of *CPU* speed: note that the number of languages in this class depends on the unit  $u$  as it is expressed in ( $\mu s$ ,  $ns$ , etc.), the choice of unit being up to the user .

To map languages  $\mathcal{L}_{u_1}(\tau_1)$  and  $\mathcal{L}_{u_2}(\tau_2)$  on  $\mathcal{L}_u(\tau_1)$  and  $\mathcal{L}_u(\tau_2)$ , we use the morphism  $\phi_{u_i}^u$ , which is defined in the following way (see Figure 10 for the use of this morphism):

**Definition 10** Let  $u \in \mathbb{N}^*$ ,  $u_i \in u\mathbb{N}^*$ ,  $\Sigma = \{a, \bullet\}$  be the alphabet associated with a job, and  $\Sigma' = \{P, V\}$  be an alphabet associated with a resource-like constraint. The morphism  $\phi_{u_i}^u$  is defined in the following way:

$$\phi_{u_i}^u : \Sigma \cup \Sigma' \rightarrow (\Sigma \cup \Sigma')^{\frac{u_i}{u}}, \begin{cases} \bullet \rightarrow \bullet \frac{u_i}{u} \\ a \rightarrow a \frac{u_i}{u} \\ P \rightarrow P a^{(\frac{u_i}{u}-1)} \\ V \rightarrow a^{(\frac{u_i}{u}-1)} V \end{cases}$$

Note that by induction,  $\phi_{u_i}^u$  not only addresses task behaviours involved by a single resource, but also those involved by many resources.

We can now give the computing process for synchronized products :

- (1) The input datas are the system  $\Gamma$  (composed of the  $\tau_i$ 's), the execution context  $C$ , the  $\mathcal{L}_{u_i}(\tau_i)$ 's, and the  $u_i$ 's.
- (2) We compute  $u = \gcd_{i \in [1, n]}(u_i)$ .
- (3) For each  $\tau_i$ , we compute the language  $\mathcal{L}_u(\tau_i)$ , which is equivalent to the language  $\mathcal{L}_{u_i}(\tau_i)$  in the sense that both languages belong to  $\mathbb{L}(\tau_i)$ . Note that by construction,  $\mathcal{L}_u(\tau_i)$  is unique.
- (4) We compute the synchronized product language  $\mathcal{L}_u\left(\frac{\Gamma}{C}\right)$ , following the techniques presented in previous sections, since all languages now share the same time unit.

## 6.2 feasibility Decision

This synchronized product language is associated with a unique time unit, and it is a regular language. Consequently, theorem 2 remains valid.

## 7 Computing techniques

The decision process is based upon both the computing of the center of the synchronized product and the evaluation of the strong feasibility (theorem 2).

Computing synchronized products is based upon classical filter techniques[10]: the synchronized product is a subset of the homogeneous product: we construct it by starting from the initial state (it is unique), and by building accessible states, i.e. states whose incoming edges are labeled with elements of the current synchronization set (according to Arnold-Nivat's technique) . To limit the size of intermediate computing automata , we use the principle *the more resources the job uses, the earlier its integration in the synchronized product*.

The *center* operation is likewise implemented through filtering techniques, which can be applied while building the automaton: one must decide, for each edge to be built, if it leads to a valid or an invalid state.

The reader can observe the details of computings using these techniques in the synchronized product trace tables for the AMADO project validation (see §3.5, §4.3 and §5.3): the consequence of this technique is the size-limitation of temporary automata. Here, we are trying to lessen this limitation by avoiding the construction of useless components belonging to the product automaton.

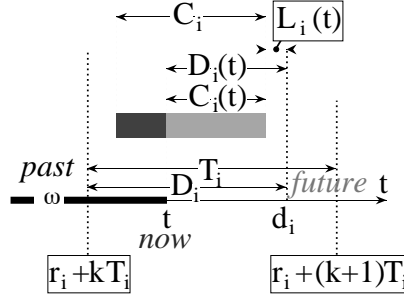


Figure 11. Dynamic time parameters of tasks

### 7.1 Improvement criterion

By studying the dynamic load of the system when in state  $s$  of the automaton, one can detect edges outgoing from  $s$  and approaching invalid components of the automaton. The goal of this section is to point out an analytic criterion to detect such edges as frequently as possible, in order to omit them in the construction of the automaton.

We deal with context  $D(\Delta)CCVT(p)$ . All reasonings involve the synchronized product automaton, not automata associated with individual jobs. Thus, they also stand for context  $D(\Delta)SCVT(p)$ , since a unique time unit is associated with the synchronized product in both frameworks.

Let us have a state  $s$  of  $A_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$ . We consider an edge  $e$ , labelled with  $x$ , outgoing from  $s$ . This edge is useless if for every word  $\alpha$  which labels a path from the initial state to  $s$ , there does not exist a word  $\beta$  such that  $\alpha x \beta \in \mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$ . Deciding this from minimal information is possible. Since generic automata associated with jobs only accept time-valid behaviours, time constraints need not be controlled when computing the synchronized product: the only constraints requiring consideration are resource synchronization (resp. message transmission). Then, we must define an analytical criterion, addressed when synchronizing with a resource, in order to compute the validity of each edge to build. The aim of this part is to define the criterion.

Recall that for  $\tau_i$ ,  $C_\tau$  is computed in terms of time units. To be compared with both the  $D_i$ ,  $T_i$ , and so on, it must be expressed in absolute time, by using the formula  $C_i u_i$ .

We note by  $T_\Gamma$  the value  $\text{lcm}_{i \in [1, n]}(T_i)$ .

We call  $L_i(t)$  the dynamic laxity of task  $\tau_i$  at time  $t$ : it is the number of possible idle time units for  $\tau_i$  before its next deadline (see Figure 11). The past of  $\Gamma$  is a behaviour between times 0 and  $t$ : it is modeled by a word  $\omega$  of  $\mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$  of length  $t$ . Thus, the laxity  $L_i(t)$  can always be computed

from  $\omega$ . That is why in the following, we note  $L_i(t)$  as well as  $L_\tau(\omega)$ . The same notation is also used for the dynamic CPU load  $C_i(t)$  (remaining CPU load for  $\tau_{ik}$  to be completed, see Figure 11).

Let  $\tau_{ik}$  be the  $k^{th}$  instance of  $\tau_i$ , presumed to be on at time  $t$ . From  $\omega$ , the dynamic laxity  $L_i(t)$  of  $\tau_{ik}$  can be obtained in the following way. By construction,  $\pi_i(\omega)$  can be broken down into  $\omega_{ba}\omega_{pi_1}\dots\omega_{pi_{k-1}}\mu$ :  $\omega_{ba}$  models the inactivity of  $\tau_i$  before its first activation, the  $\omega_{pi_j}$ 's its past completed instances, and  $\mu$  the beginning of the ongoing instance  $\tau_{ik}$ . We consequently obtain  $\forall j \in [1, k-1], |\omega_{pi_j}| = T_i$  and  $|\mu| \leq T_i$ . There exists a set  $\Lambda$  of words such that  $\nu \in \Lambda \Rightarrow |\mu\nu| = T_i \wedge \pi_i(\omega)\nu \in \mathcal{L}_u(\tau_i)$ .  $L_i(\omega)$  is obtained as  $\underset{\nu \in \Lambda}{Min}(|\nu|_\bullet)$ : it is the most constrained dynamic laxity of  $\tau_{ik}$  induced by the scheduling sequence  $\omega$ .

We note by  $\Sigma_i$  the alphabet associated with  $\mathcal{L}_u(\tau_i)$ . A word  $\beta$  of  $\prod_{i=1}^{i=n+r} \Sigma_i$  is valid (and then must not be filtered by the criterion) if  $\omega\beta \in \mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$ : if this property stands, the behaviour  $\omega\beta$  leads  $\Gamma$ , at time  $t + |\beta|$ , to reach the activation of the  $k+1^{th}$  instance of  $\tau_i$  according to both time and resource constraints. If not, choosing  $e$  leads to a time fault:  $e$  may be omitted in the construction of  $A_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right)$ .

[13] gives such a criterion for the context  $D(\Delta)CCT(1)$ . It uses the order  $\prec_\omega$  on  $\Gamma$ , which is defined in the following way: at time  $t$  (i.e. at the end of  $\omega$ ),  $\tau_i \prec_\omega \tau_j \Leftrightarrow L_i(\omega) < L_j(\omega)$ . The criterion is defined by:

$$\omega\beta \in \mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(1)}\right) \Rightarrow \forall \tau_i \in \Gamma, \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_\omega \tau_i}} (C_j(\omega\beta)) \leq \frac{L_i(\omega\beta)}{u}$$

At this point, we extend this criterion to the context  $D(\Delta)CCVT(p)$ .

We obtain:

### Theorem 3

$$\omega\beta \in \mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(p)}\right) \Rightarrow \forall \tau_i \in \Gamma, \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_\omega \tau_i}} \frac{C_j(\omega\beta)}{p} \leq \frac{L_i(\omega\beta)}{u}$$

### Proof

We enrich the notations we consider for context, by including time units in their descriptions: we note  $D(\Delta)CCT(p[u])$  to indicate the (common) time unit associated with the  $p$  processors of the target system.

Let  $C_p$  be a  $D(\Delta)CCT(p[u])$  configuration,  $\omega \in \mathcal{L}_u\left(\frac{\Gamma}{D(\Delta)CCVT(p[u])}\right)$  and



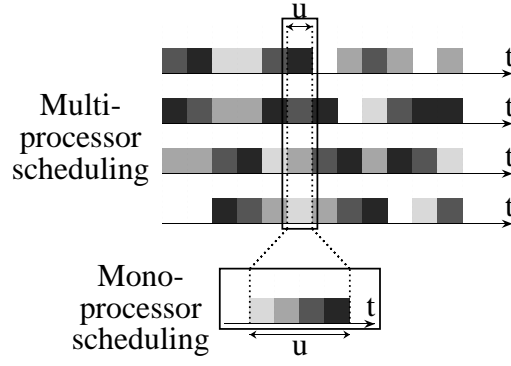


Figure 12. Equivalent mono-processor model

$$\beta \in \prod_{i=1}^{i=n} \Sigma_i, \text{ such that } \omega\beta \in \mathcal{L}_u \left( \frac{\Gamma}{D(\Delta)CCVT(p[u])} \right).$$

We consider a configuration  $C_1$  following  $D(\Delta)CCT\left(1\left\lceil\frac{u}{p}\right\rceil\right)$ : its processor is  $p$  times faster than the  $p$  processors of  $C_p$ . The load capacities of  $C_1$  and  $C_p$  are identical. There exist  $\xi \in \mathcal{L}_p \left( \frac{\Gamma}{D(\Delta)CCVT(1)} \right)$  and  $\beta' \in \prod_{i=1}^{i=n} \Sigma_i$  such that (see Figure 12)

$$\left\{ \begin{array}{l} |\xi| = p \times |\omega| \\ \forall i \in [1, n], \pi_{-\bullet}(\pi_i(\omega)) = \pi_{-\bullet}(\pi_i(\xi)) \\ |\beta'| = p \\ \forall i \in [1, n], \pi_{-\bullet}(\pi_i(\beta)) = \pi_{-\bullet}(\pi_i(\beta')) \\ \forall k \in [1, |\omega|] \left\{ \begin{array}{l} \pi_i(\omega_k) \neq \bullet \Leftrightarrow \pi_i(\omega_k) = \pi_i(\pi_{-\bullet}(\xi_{(k-1) \times p + 1} \dots \xi_{k \times p})) \\ \pi_i(\omega_k) = \bullet \Leftrightarrow \pi_i(\pi_{-\bullet}(\xi_{(k-1) \times p + 1} \dots \xi_{k \times p})) = \epsilon \end{array} \right. \end{array} \right.$$

The correlation between duration and length leads  $L_i(\omega\beta)$  to be equal to  $L_i(\xi\beta')$ , for each  $\tau_i \in \Gamma$ , and  $\preceq_\omega$  to be equivalent to  $\preceq_\xi$ . Then, since  $\forall \tau_i \in \Gamma, uC_i(\omega) = p \times \frac{u}{p}C_i(\xi)$ , we obtain  $\sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_\xi \tau_i}} \frac{u}{p} (C_j(\xi\beta')) = \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_\omega \tau}} \left( \frac{uC_i(\omega\beta)}{p} \right).$

Since  $\omega\beta \in \mathcal{L}_u \left( \frac{\Gamma}{D(\Delta)CCVT(p[u])} \right)$ , we have  $\forall x \in \mathbb{N}, \exists \alpha \in \left( \prod_{i=1}^{i=n} \Sigma_i \right)^x \left( \prod_{i=1}^{i=n} \Sigma_i \right)^*$  such that  $\omega\beta\alpha \in \mathcal{L}_u \left( \frac{\Gamma}{D(\Delta)CCVT(p[u])} \right)$ . By a construction such as that of  $\xi$  and  $\beta'$ , we can build a word  $\alpha'$  of length  $x$  such that  $\xi\beta'\alpha' \in \mathcal{L}_p \left( \frac{\Gamma}{D(\Delta)CCT(1\lceil\frac{u}{p}\rceil)} \right)$ . As a result,  $\xi\beta'$  belongs to the center of the mono-processor language, and the

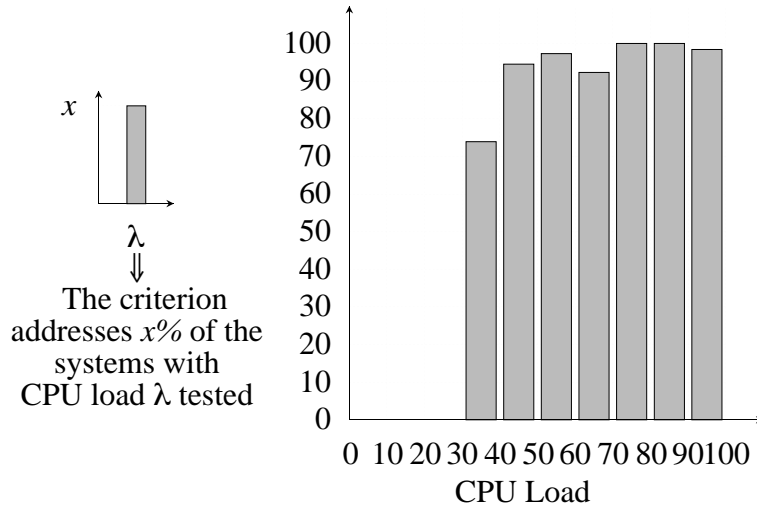


Figure 13. Criterion base

mono-processor criterion stands. Then, we obtain

$$\forall \tau_i \in \Gamma, \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_{\xi} \tau_i}} \left( \frac{u}{p} C_j(\xi\beta') \right) \leq L_i(\xi\beta')$$

Since  $\sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_{\xi} \tau_i}} \frac{u}{p} (C_j(\xi\beta')) = \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_{\omega} \tau_i}} \left( \frac{u C_j(\omega\beta)}{p} \right)$ , we obtain

$$\forall \tau_i \in \Gamma, \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_{\omega} \tau_i}} \left( u \frac{C_j(\omega\beta)}{p} \right) \leq L_i(\omega\beta)$$

and then

$$\forall \tau_i \in \Gamma, \sum_{\substack{\tau_j \in \Gamma \\ \tau_j \preceq_{\omega} \tau_i}} \left( \frac{C_j(\omega\beta)}{p} \right) \leq \frac{L_i(\omega\beta)}{u}$$

□

## 7.2 Experimental analysis

Let us now evaluate the improvement brought about by this multi-processor branching-bound criterion. This evaluation is performed by applying the operational validity decision process with and without the use of the criterion on a random sample  $\xi$  generated thanks to a real-time configuration random generator, which is designed to produce configurations to implement on multi-processor targets. The generator we have used was proposed by J.Goossens and C.Marq in [18]. Since operational validation is connected with CPU loads, this sample is analysed by CPU load sections. For a real-time system  $\Gamma = (\tau_i)_{i \in [1, n]}$

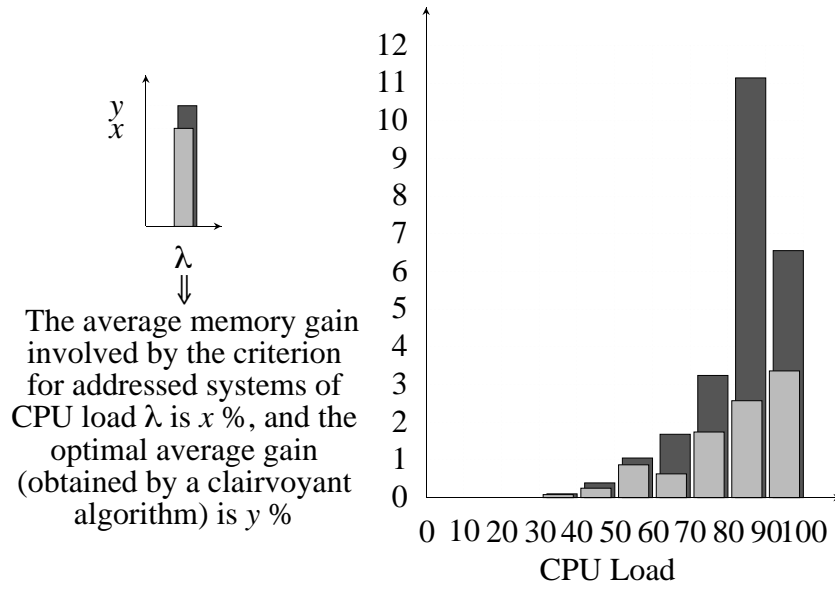


Figure 14. Memory gain involved by the criterion

designed to run under the context  $D(\Delta)CCT(p)$ , the CPU load is  $\frac{\sum_{i=1}^{i=n} \frac{C_i}{T_i}}{p}$ . For each CPU load  $\lambda \in [0, 1]$ , we call  $\xi_\lambda$  the subset of  $\xi$  composed of configurations whose CPU load is  $\lambda$ . We note  $\xi_{[\lambda_1, \lambda_2]}$  the set  $\bigcup_{\lambda \in [\lambda_1, \lambda_2]} \xi_\lambda$ . We do not deal with configurations with CPU loads of less than 0.3, because they are of negligible significance in a real-time context.

The random sample  $\xi$  has been generated from a CPU load uniform random law on the interval  $[\frac{3}{10}, 1]$ . Then,  $\xi$  is partitioned in  $\left\{ \left( \xi_{[\frac{i-1}{10}, \frac{i}{10}]} \right)_{i \in [4, 10]} \right\}$ , and each  $\xi_{[\frac{i-1}{10}, \frac{i}{10}]}$  contains around 200 configurations. On the bar graphs presented in Figures 13, 14 and 15, each presented value corresponds to the average value for the corresponding sample  $\left\{ \left( \xi_{[\frac{i-1}{10}, \frac{i}{10}]} \right)_{i \in [4, 10]} \right\}$ .

We evaluate the criterion firstly on its base aspect (is it often useful?), and secondly on its improvement aspect (does it bring about a real improvement, in both space and time complexity aspects?).

We get the following results:

- Firstly, we evaluate the criterion base (see Figure 13): for each sample  $\zeta$  of  $\left\{ \left( \xi_{[\frac{i-1}{10}, \frac{i}{10}]} \right)_{i \in [4, 10]} \right\}$ , we compute the operational validity decision for all configurations of  $\zeta$ . While computing, we enumerate the  $p$  configurations whose operational validity decision computing is improved by the use of the criterion: the value  $x\%$  presented on the figure is  $\frac{p}{|\zeta|}$ .

Observing Figure 13 shows that the criterion is nearly always useful for real-time addressed configurations (CPU loads over 70%).

- Secondly, the main problem of model checking techniques in such studies is space explosion. Such criteria are usually followed in order to improve both space and time complexities. In Figure 14, we evaluate space gain.

This gain is computed from the space used by the operational validity decision algorithm: computing automata for each job, making products and intersections, computing the center language accepting automaton, evaluating the emptiness of this center language by analysing this automaton. For  $C \in \zeta$ , let us call  $bspace(C)$  the space complexity (i.e. the larger instantaneous amount of memory needed by the decisional algorithm) for obtaining the validity decision following this technique. Now, let us call  $cspace(C)$  the space complexity of the algorithm when it is upgraded with the criterion, and let us call  $ospace(C)$  the space complexity of a clairvoyant algorithm, which built only the edges that remain in the final automaton.

In Figure 14, we present the average values, for the ten sample  $\xi_{[\frac{i-1}{10}, \frac{i}{10}]}$  previously described, of both  $1 - \frac{cspace(C)}{space(C)}$  (they compose the bright histogram) and  $1 - \frac{ospace(C)}{space(C)}$  (they compose the dark histogram). These two histograms provide a comparison between the criterion and the optimal possible performance.

We can see that the clairvoyant algorithm never deletes more that about 10% of built edges. In Section 3, we have designed our model in order to collect only valid scheduling sequences. By only improving our basic space complexity about 10% for the worst case section, and by about 5% on the others, the clairvoyant algorithm both validates our approach and yields improvements that would otherwise be hard to obtain. However, we see on Figure 14 that our criterion improves space complexity about 3% (for addressed configurations), that is to say about 50% of the clairvoyant algorithm capacities (the optimum). However, for Section [80%, 90%], the criterion improves only about 25% with regard to the optimum. This case can be explained in the following way: such configurations are not loaded enough to often be invalid, but are loaded enough to lead to deadline missings generated by markedly earlier scheduling decisions. Such time properties lead the center automaton computing to delete a great part of the graph: this is precisely the scope of improvement of the criterion, so it is natural that having to process a larger part of the graph than in other sections, it loses efficiency against the clairvoyant algorithm.

- Thirdly, evaluating the criterion uses processor time in addition to basic algorithm processor time. Here, we cannot consider the clairvoyant algorithm as a reference, since such an algorithm does not exist (because of the complexity class of the problem). That is why, on Figure 15, we evaluate only the time gain involved by use of the algorithm: in many cases, the additional CPU time involved by the criterion is made up for by the edge gain obtained: the only section to be in deficit is [30%, 40%]. This deficit stands

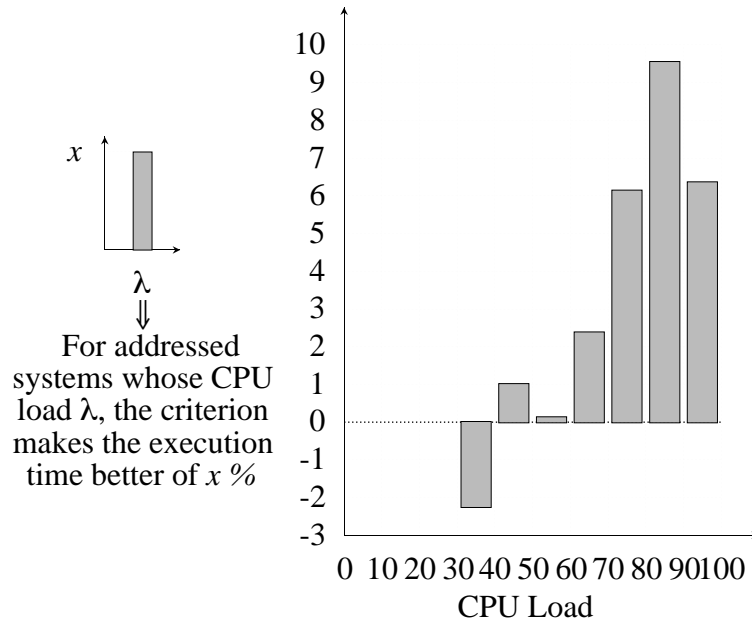


Figure 15. Time gain involved by the criterion

because for such systems, the basic algorithm is very efficient (see Figure 14), and then the additional expense involved by computing the criterion for each edge is not made up for by avoiding edges. For addressed sections, the time-gain is over 6%, and about 10% for the Section [80%, 90%]. Recall that space improvement of the criterion for this section is only about 2.5% (see Figure 14), but these 2.5% are connected with a time gain about 10%: the best of all sections.

So, these experimentations show that our model achieves its objectives, since it built only a few useless edges, and that the criterion is definitely useful for configurations addressed in real-time conception processes.

## 8 Conclusion

In this paper, we have used algebraic compositions of regular languages to define a model to decide upon the operational validity of periodic real-time systems. This model is valid for periodic real-time systems with offsets, where jobs can communicate or share critical resources, and it takes hardware specifications into account (e.g. processors of different speeds).

Experiments have shown that this model is quite efficient: the validity decision process built very few useless edges, and the branching-bound criterion reduces decision execution times by 5% to 10%. Moreover, the AMADO project study shows that this approach is useful for Real-Time Conception Aid Design. The classes of properties that can be evaluated are currently being studied.

This work yields many contributions pertaining to the real-time operational validation problem:

- Reducing job CPU loads to their worst case is known to lead to acceptance of invalid configurations. We have provided a novel methodology which avoids worst case analysis in the operational validation process.
- This methodology applies to mono-processor target architectures, but (and this is new) also to multi-processors.
- Since we represent the set of valid sequences with regular languages, the cyclicity theorem [25][13] remains valid for multi-processor scheduling.

The second step of this research consists in defining and experimenting measure indicators useful in software quality assessment. We are presently working in two directions:

- we are characterizing the classes of indicators we can compute on the basis of our model: from a technical point, we are comparing the respective performances of enumeration-based indicators [33] and of probability-based indicators [30][28] for their computing time as well as for their expressivity powers;
- we are characterizing, in the automaton, the paths corresponding to RM scheduling sequences (resp. DM, EDF, LF): measure indicators will be useful in determining how a real-time system needs to be modified so as to be rendered valid for a specific on-line policy.

Moreover, our results encourage us to further the approach we have studied to more general systems: we have extended the scope of this methodology to real-time systems composed of both periodic and sporadic jobs [17]: a sporadic job is associated with an alarm signal, which is obviously not periodic. At another location, observation of the geometric properties of the automata's graphs has prompted us to adopt a new approach toward validating real-time, and it appears to be highly efficient [24]. At some future time, we are planning to apply these results so as to provide assistance for real-time conceivers in the operational specification process.

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## Appendix: Notations and Definitions used in this paper

### Mathematical notations

|                         |  |
|-------------------------|--|
| $\prod_{i \in I} E_i$   | Cartesian product of all sets $E_i$ such that $i \in I$  |
| $\bigcup_{i \in I} E_i$ | Union of all sets $E_i$ such that $i \in I$  |
| $\bigcap_{i \in I} E_i$ | Intersection of all sets $E_i$ such that $i \in I$   |
| $A \setminus B$         | $\{x \in A \cup B \text{ such that } x \in A \wedge x \notin B\}$  |
| $\pi_B \in B^A$         | Function such that $x \in B \rightarrow \pi_B(x) = x$ and $x \notin B \rightarrow \pi_B(x) = \epsilon$               |
| $\pi_{\neg B} \in B^A$  | Function such that $x \in B \rightarrow \pi_{\neg B}(x) = \epsilon$ and $x \notin B \rightarrow \pi_{\neg B}(x) = x$ |

### Words and languages

|                         |  |
|-------------------------|--|
| $ \omega $              | Length (number of characters) of word $\omega$                 |
| $ \omega _x$            | Number of occurrences of pattern $x$ in the word $\omega$      |
| $Reg(\Sigma)$           | Set of regular languages on alphabet $\Sigma$                  |
| $Pref(\omega)$          | Set of prefixes of $\omega$ (can be a word or a language)      |
| $Center(L)$             | Center of the language $L$                                     |
| $a^x$                   | The pattern $a$ repeated $x$ times                             |
| $\bigcup_{i \in I} L_i$ | Homogeneous product of all languages $L_i$ such that $i \in I$ |

**Real-time tasks**

|             |   |
|-------------|---|
| $\Gamma$    | the currently considered real-time system |
| $\tau_i$    | The $i^{th}$ task of a real-time system   |
| $\tau_{ij}$ | The $j^{th}$ instance of task $\tau_i$    |
| $r_i$       | First activation date of task $\tau_i$    |
| $D_i$       | Critical delay of task $\tau_i$           |
| $C_i$       | CPU load of task $\tau_i$                 |
| $T_i$       | Period of task $\tau_i$                   |
| $L_i(t)$    | Dynamic laxity of task $\tau_i$           |
| $C_i(t)$    | Dynamic CPU load of task $\tau_i$         |

**Scheduling contextes**

|                   |   |                             |
|-------------------|---|-----------------------------|
| $ICCT(p)$         | - Independent tasks                         | - Periodic real-time system |
|                   | - Common clock                              | - $p$ processors            |
|                   | - Centralized system                        |                             |
| $D(\Delta)CCT(p)$ | - Dependent tasks with constraints $\Delta$ | - $p$ processors            |
|                   | - Common clock                              | - Periodic real-time system |
|                   | - Centralized system                        |                             |
| $EDF$             | Earliest Deadline First scheduling policy   |                             |
| $LL$              | Least Laxity First scheduling policy        |                             |
| $RM$              | Rate Monotonic scheduling policy            |                             |
| $DM$              | Deadline Monotonic scheduling policy        |                             |

**Model**

|                                   |  |
|-----------------------------------|--|
| $a$                               | The task is running for one observation time unit                                  |
| $\bullet$                         | The task is suspended for one observation time unit                                |
| $\mathcal{L}(\frac{\Gamma}{C})$   | Model language for system $\Gamma$ under context $C$                               |
| $A(\frac{\Gamma}{C})$             | Acceptation finite automaton for $\mathcal{L}(\frac{\Gamma}{C})$                   |
| $\mathcal{L}_u(\frac{\Gamma}{C})$ | Model language for system $\Gamma$ under context $C$ and observation time unit $u$ |
| $A_u(\frac{\Gamma}{C})$           | Acceptation finite automaton for $\mathcal{L}_u(\frac{\Gamma}{C})$                 |
| $S_R$                             | Arnold-Nivat synchronization set for resource $R$ management protocol              |

$\phi_n^p$  Transformation function which replaces  $a$  by  $a^{\frac{n}{p}}$ ,  $P$  by  $Pa^{\left(\frac{n}{p}-1\right)}$  and  $V$  by  $a^{\left(\frac{n}{p}-1\right)}V$ .  $p$  must be element of  $n\mathbb{N}$